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Isotopic dependence of charge and matter radii.

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NUCLEI
Theory

Self-Consistent Calculation of the Charge Radii in the $^{58-82}$Cu Isotopic Chain

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NUCLEI
Theory

Self-Consistent Study of Nuclear Charge Radii in Ar–Ti Region

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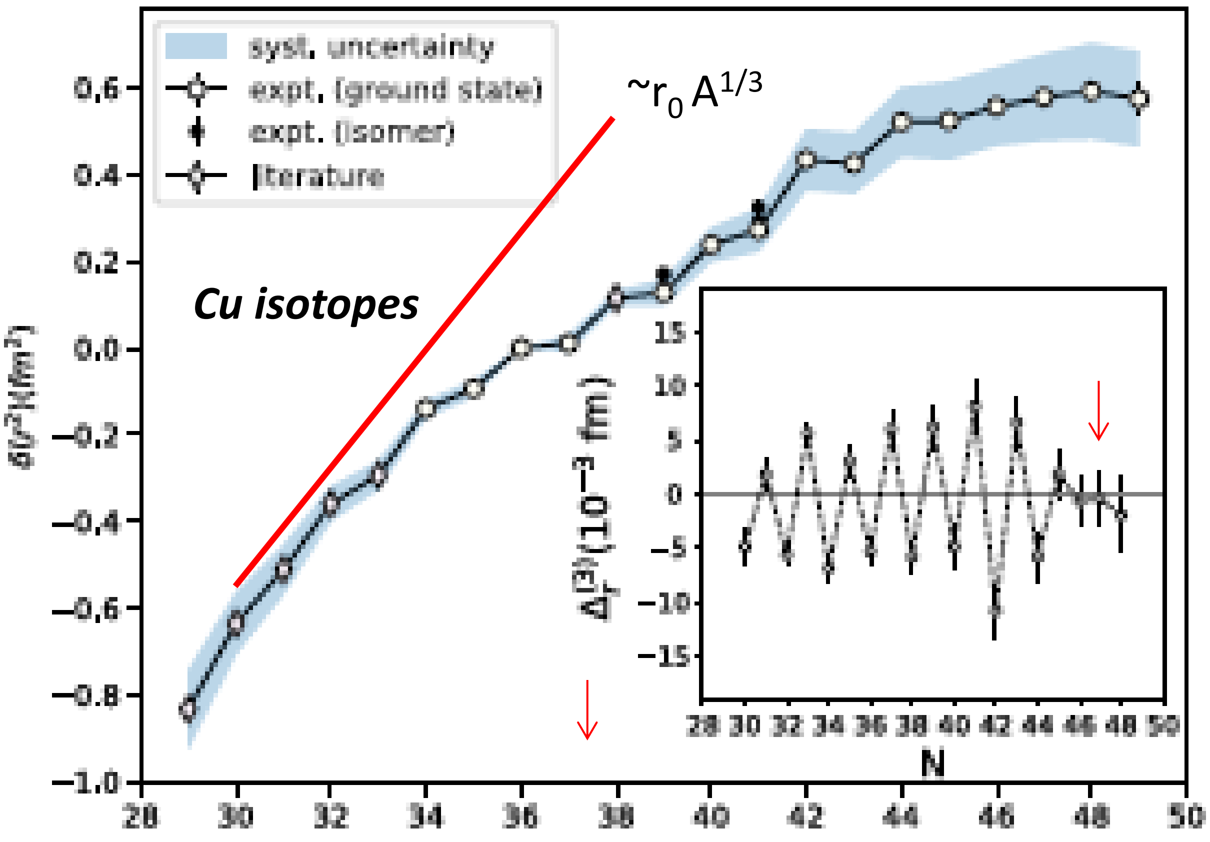
Isotopic dependence of charge radii.

Особая роль принадлежит зарядовым ядерным радиусам, содержащим информацию о мезоскопической природе атомных ядер. Глобальные закономерности изменения размеров ядер (их протонного нейтронного, массового или зарядового радиусов) с ростом массового числа описываются простой жидкокапельной формулой $R = r_0 A^{1/3}$, следующей из макроскопического подхода. Однако экспериментально наблюдается другая, более слабая A-зависимость, приводящая к “отрицательному” (по сравнению с жидкокапельным) изотопическому сдвигу радиусов. Более того, наблюдаются заметные локальные флуктуации зарядовых радиусов. Это специфически квантовое явление, отражающее эволюцию оболочечной структуры ядер. Наиболее яркие примеры нерегулярностей изотопической зависимости зарядовых радиусов — локальные минимумы (или максимумы) при пересечении замкнутых оболочек и четно-нечетное “дрожание” радиусов (odd-even staggering или OES-эффект) [3].



de Groote, R.P., Billowes, J., Binnersley, C.L. *et al.* Measurement and microscopic description of **odd–even staggering** of charge radii of exotic copper isotopes. [***Nature Physics* 16, 620–624**](#) (2020).

$$\Delta(3)_r \sim R(N-1) - 2 * R(N) + R(N+1)$$

$$R_{ch} = \sqrt{\langle r_{ch}^2 \rangle}.$$

$$< r_{ch}^2 >= \frac{1}{Z} \int r^2 \rho_{ch}(\mathbf{r}) d^3 r \,.$$

$$R_{ch} = \sqrt{\langle r_{ch}^2 \rangle}.$$

If the isotope shift measurements are performed on more than one isotope, one can find the difference in the hyperfine structure centroid frequency of two isotopes with mass numbers A and A′.

The changes in the mean-square charge radii $\delta\langle r^2 \rangle$ are calculated from the isotope shift $\delta\nu$ (A, A′) via

$$\delta\langle r^2 \rangle = \frac{1}{F}[\nu^{AA'} - (K_{NMS} + K_{SMS}) \frac{m_A - m_{A'}}{(m_A + m_e) m_{A'}}].$$

Here F, K_{SMS} and K_{NMS} are the ***atomic field shift, specific mass shift and normal mass shift*** *factors, respectively.*

DMS: $\delta< r_{ch}^2(A) > = \delta< r_{ch}^2(A) > - < r_{ch}^2(A') >$

$\Delta R_{ch}(3) =(-1)^{N+1}/2 \; [R_{ch}(N-1) – 2^* R_{ch}(N) + R_{ch}(N+1)]$

Collinear resonance ionization laser spectroscopy. CRIS-CERN 2016 – 2022. Nuclear charge radii.

Not described with Skyrme functionals :

Parabolic shape R(N=20) = R(N=28) . OES .

Unexpected grows of Rch at N>28 in Ca.

Fayans : DF3, FaNDF0 *(© Kurchatov Inst., 90s)*

Parabolic and OES effects are well described

*S. Fayans, S. Tolokonnikov, E. Trykov, D. Zawischa,Nucl. Phys. A676, 49 (**2000**).*

S. Tolokonnikov, I.N. Borzov, M. Kortelainen, Yu.S. Lutostansky, E.E. Saperstein J.Phys G42, 075102, 2015 First applications of Fayans functional to deformed nuclei - HFBTHO

…"recently developed " Fy (Δr; HFB)

P.-G. Reinhard and W. Nazarewicz, Phys. Rev. C 95, 064328 (2017). "Toward a global description of nuclear charge radii: Exploring the Fayans energy density functional."

The form of volume, surface and pairing parts of Fy were taken the same as in original Fayans functional. Parametrization protocol differs !

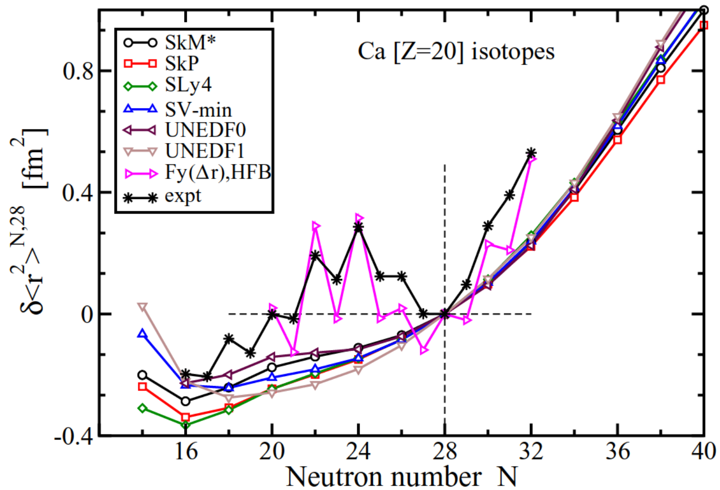
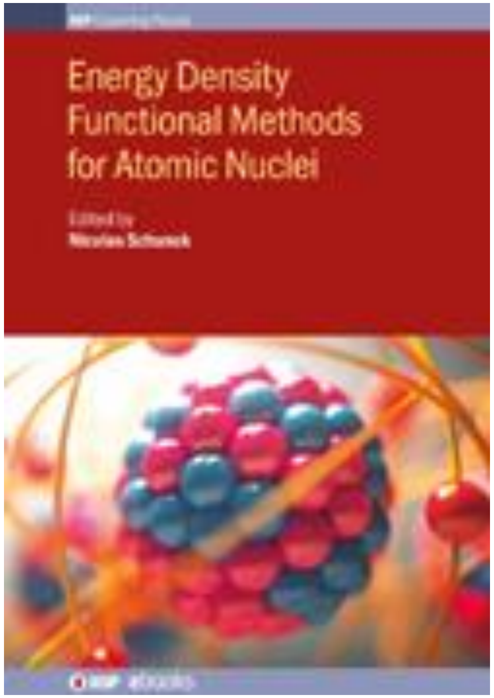


FIG. 24. The $\delta\langle r^2\rangle^{N,28}$ values of the Ca ($Z = 20$) isotopes relative to the $N = 28$ isotope. The experimental data for the Ca isotopes are mostly taken from Ref. [10] while that for the ^{39,41}Ca isotopes from Ref. [80] and for ^{36,37,38}Ca from Ref. [11]. The results of the Skyrme DFT calculations are taken from Mass Explorer at FRIB [36]. The results of the Fayans Fy(Δr, HFB) functional are taken from Fig. 4 of Ref. [30].

*Experiment : R. F. Garcia Ruiz ..... P.-G. Reinhard and W. Nazarewicz…. et al., **Nature Physics**, 12, 594 (2016). Unexpectedly large charge radii of neutron-rich calcium isotopes*

U. C. Perera ,A. V. Afanasjev , P. Ring

*PHYSICAL REVIEW C **104**, 064313 (2021).*



Phenomenological EDF

DF3… -a, -b, -f ,…FANDF^0
S.A. Fayans + collaborators, KI ,Moscow

BCPM - *Barcelona–Catania–Paris–Madrid*
(originating from an early work by Baldo et al.)

SeaLL - *Seattle–Livermore .*

- *directly parametrize the nuclear EoS by series of powers of the density ;*
- *terms accounting for finite-size and many-body effects .*

- *Fayans functional: nuclear correlation term in Coulomb exchange*
- *Density gradient pairing .*

Fayans and SeaLL functionals are the Kohn-Sham type EDF :
free (independent-particle) kinetic energy operator

$$\tau = p^2/2M, \quad m^*/M = 1$$

Self-Consistent Ground State. Fayans EDF.

$$\mathcal{E}[\rho(\mathbf{r}),\nu(\mathbf{r})] = \tau + \varepsilon_v + \varepsilon_s + \varepsilon_{\text{Coul}} + \varepsilon_{sl} + \varepsilon_{ss} + \varepsilon_{\text{pair}} \,.$$

$$E_0^{\text{int}}[\rho] = \int \mathcal{E}(\rho(\mathbf{r}))d^3r = \int \frac{a\rho^2}{2}\left(1+\alpha\rho^\sigma\right)d^3r,$$

Skyrme EDF

$$\mathcal{E}(\rho) = \frac{a\rho^2}{2}\frac{1+\alpha\rho^\sigma}{1+\gamma\rho}.$$

E_vol, E surf : ρ – dependent terms of Fayans EDF

Fractional (Pade-like) ansatz allows for:

- *transformation of the EDF components to Migdal quasiparticles;*
- *Besides that one can retrieve the volume EDF parameters*

$$a_\pm^{\rm v},\, h_{1\pm}^{\rm v},\, h_{2\pm}^{\rm v} \leftrightarrow E/A_{\rm eq},\, \rho_{\rm eq},\, K,\, J,\, L,\, h_{2-}^{\rm v}$$

from symmetric nuclear matter EOS (Fridman-Panharipande)

Most of the nuclear EDFs used in self-consistent mean-field calculations have been derived from phenomenological effective interactions. The Fayans functional differs from Skyrme functional in the volume, surface and pairing parts .

$$\mathcal{E} = \mathcal{E}^{\mathrm{v}}(\rho,\tau) + \mathcal{E}^{\mathrm{s}}(\rho) + \mathcal{E}^{\mathrm{ls}}(\rho,\vec{J}) + \mathcal{E}^{\mathrm{Coul}}(\rho) + \mathcal{E}^{\mathrm{pair}}(\rho) + \mathcal{E}^{\mathrm{c.m.}}(\rho) \quad .$$

	Skyrme	Fayans
volume:	$\mathcal{E}^{\mathrm{v}}_{\mathrm{Sk}} = \sum_{t=0}^{1} \left[(C^{\rho\rho}_{t0} + C^{\rho\rho}_{t\mathrm{D}}\rho_0^{\alpha})\rho_t^2 + C^{\rho\tau}_t \rho_t\tau_t\right]$	$\mathcal{E}^{\mathrm{v}}_{\mathrm{Fy}} = \frac{1}{3}\varepsilon_F\rho_{\mathrm{sat}}\left[a^{\mathrm{v}}_{+}\frac{1-h^{\mathrm{v}}_{1+}x_0^{\sigma}}{1+h^{\mathrm{v}}_{2+}x_0^{\sigma}}x_0^2 + a^{\mathrm{v}}_{-}\frac{1-h^{\mathrm{v}}_{1-}x_0}{1+h^{\mathrm{v}}_{2-}x_0}x_1^2\right]$
	$C^{\rho\rho}_{t0}, C^{\rho\rho}_{t\mathrm{D}}, \alpha, C^{\rho\tau}_t \leftrightarrow E/A_{\mathrm{eq}},\ \rho_{\mathrm{eq}},\ K,\ J,\ L,\ \frac{m^*}{m},\ \kappa_{\mathrm{TRK}}$	$a^{\mathrm{v}}_{\pm},\ h^{\mathrm{v}}_{1\pm},\ h^{\mathrm{v}}_{2\pm} \leftrightarrow E/A_{\mathrm{eq}},\ \rho_{\mathrm{eq}},\ K,\ J,\ L,\ h^{\mathrm{v}}_{2-}$
surface :	$\mathcal{E}^{\mathrm{s}}_{\mathrm{Sk}} = \sum_{t=0}^{1} C^{\rho\Delta\rho}_t \rho_t\Delta\rho_t$	$\mathcal{E}^{\mathrm{s}}_{\mathrm{Fy}} = \frac{1}{3}\varepsilon_F\rho_{\mathrm{sat}}\frac{a^{\mathrm{s}}_{+}r_s^2(\vec{\nabla}x_0)^2}{1 + h^{\mathrm{s}}_{+}x_0^{\sigma} + h^{\mathrm{s}}_{\nabla}r_s^2(\vec{\nabla}x_0)^2}$
spin-orbit:	$\mathcal{E}^{\mathrm{ls}}_{\mathrm{Sk}} = \sum_{t=0}^{1} C^{\rho\nabla J}_t \rho_t \boldsymbol{\nabla}\cdot\boldsymbol{J}_t$	$\mathcal{E}^{\mathrm{ls}}_{\mathrm{Fy}} = \sum_{t=0}^{1} C^{\rho\nabla J}_t \rho_t \boldsymbol{\nabla}\cdot\boldsymbol{J}_t$
pairing:	$\mathcal{E}^{\mathrm{pair}}_{\mathrm{Sk}} = \frac{1}{4}\sum_{q\in\{p,n\}} V_{\mathrm{pair},q}\left(1 - \frac{\rho_0}{\rho_{\mathrm{pair}}}\right)\breve{\rho}_q^2$	$\mathcal{E}^{\mathrm{pair}}_{\mathrm{Fy}} = \frac{2\varepsilon_F}{3\rho_{\mathrm{sat}}}\breve{\rho}_q^2\left[f^{\xi}_{\mathrm{ex}} + h^{\xi}_{+}x^{\gamma}_{\mathrm{pair}} + h^{\xi}_{\nabla}r_s^2(\vec{\nabla}x_{\mathrm{pair}})^2\right]$

where $x_t = \rho_t/\rho_{\mathrm{sat}}$ and $x_{\mathrm{pair}} = \breve{\rho}_q/\rho_{\mathrm{sat}}$. The γ, $\rho_{\mathrm{sat}} = 0.16\ \mathrm{fm}^{-3}$ and $\varepsilon_F = \varepsilon_F(\rho_{\mathrm{sat}})$ are given, fixed values. The non-linear surface coefficient is fixed as $h^s_+ = h^v_{2+}$. Coulomb term and c.m. correction are irrelevant here. Note that the parameters for the volume terms are handled in term of nuclear matter parameters E/A_{eq} etc as is indicted in the line below the volume terms.

V.A Khodel, E.E Saperstein Phys.Repts. 92 (1982) ,
A.B Migdal Finite Fermi-System Theory. 2nd ed. , Nauka, Moscow, 1983,
S.A Fayans JETP Letters 104 (1998)

Pairing EDFs - depends on ρ , grad(ρ).

$$\varepsilon_{\text{pair}}(\mathbf{r}) = \frac{1}{2}\sum_{\tau = n,p} \mathcal{F}^{\xi,\tau}(\rho_+(\mathbf{r}))|\nu_\tau(\mathbf{r})|^2.$$

$$f^\xi(x_+(\mathbf{r})) = f^\xi_{\text{ex}} + h^\xi(x_+)^q(\mathbf{r}) + f^\xi_\nabla r_0^2 (\nabla x_+(\mathbf{r}))^2 \ .$$

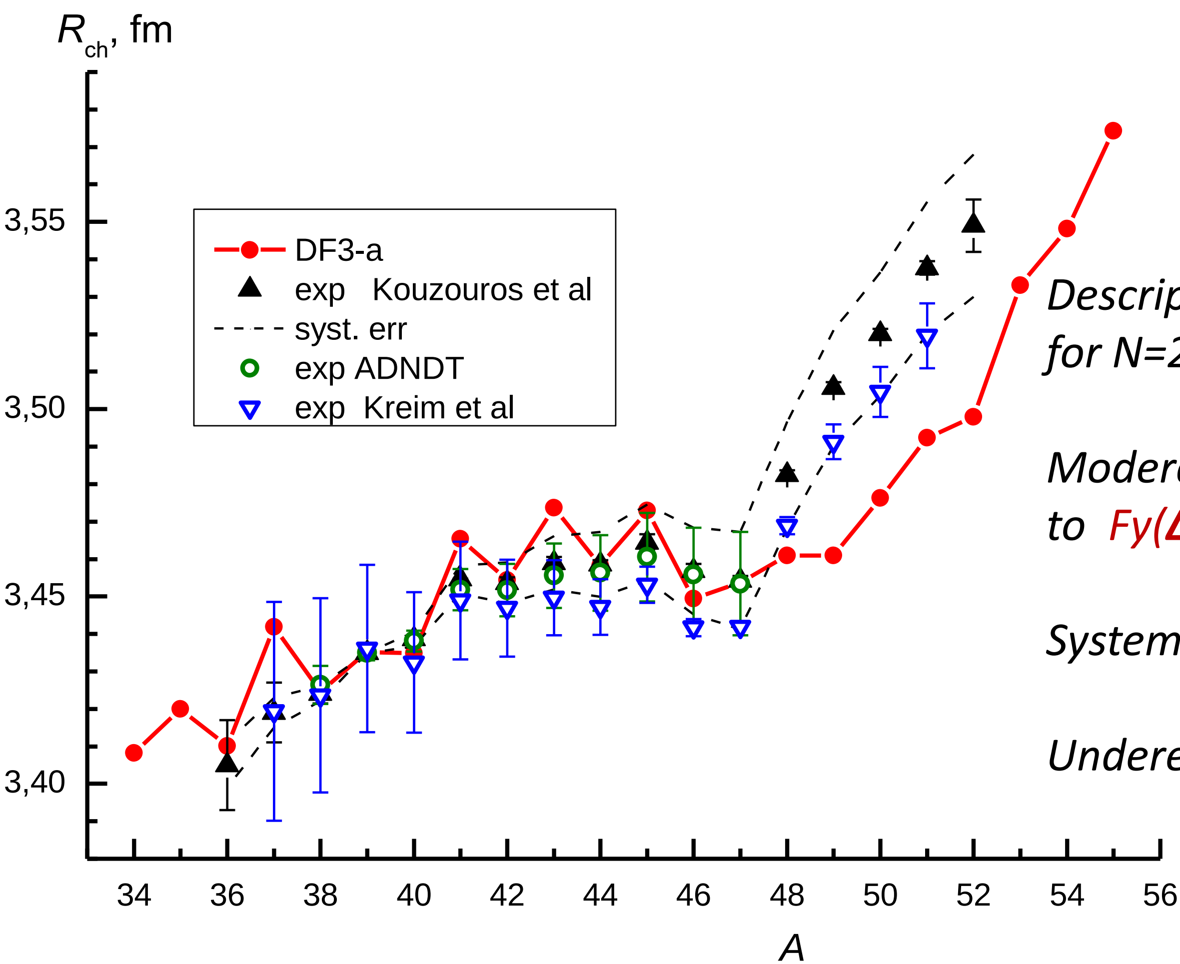
$x_+ = (\rho_p + \rho_n)/ 2\, \rho_0$

*Fy (**Δr**, HFB)*

*P.-G. Reinhard and W. Nazarewicz, Phys. **Rev. C 95, 064328 (2017)***

The pairing parts taken from the Fayans functional.

Rch (rms) Potassium isotopes



Description of the parabolic shape for N=20 -28 is reasonable.

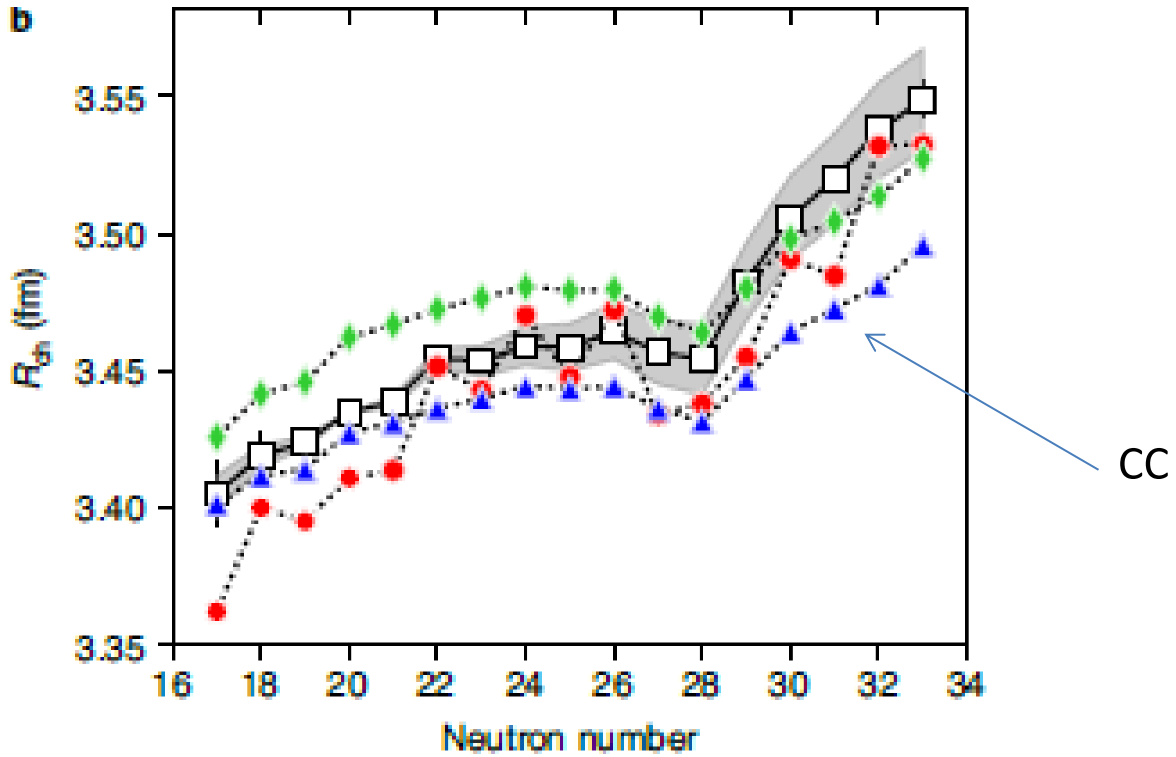
*Moderate OES at N>28 in contrast to Fy(**Δr**, HFB)*

Systematical errors @ N~32 are high

Underestimate at N>28 for K isotopes .

Black : Á. Koszorús et.al., Nat.Physics, doi.org/10.1038/s41567-020-01136-5

Experimental and Fy (Δ, BCS) rms charge radii

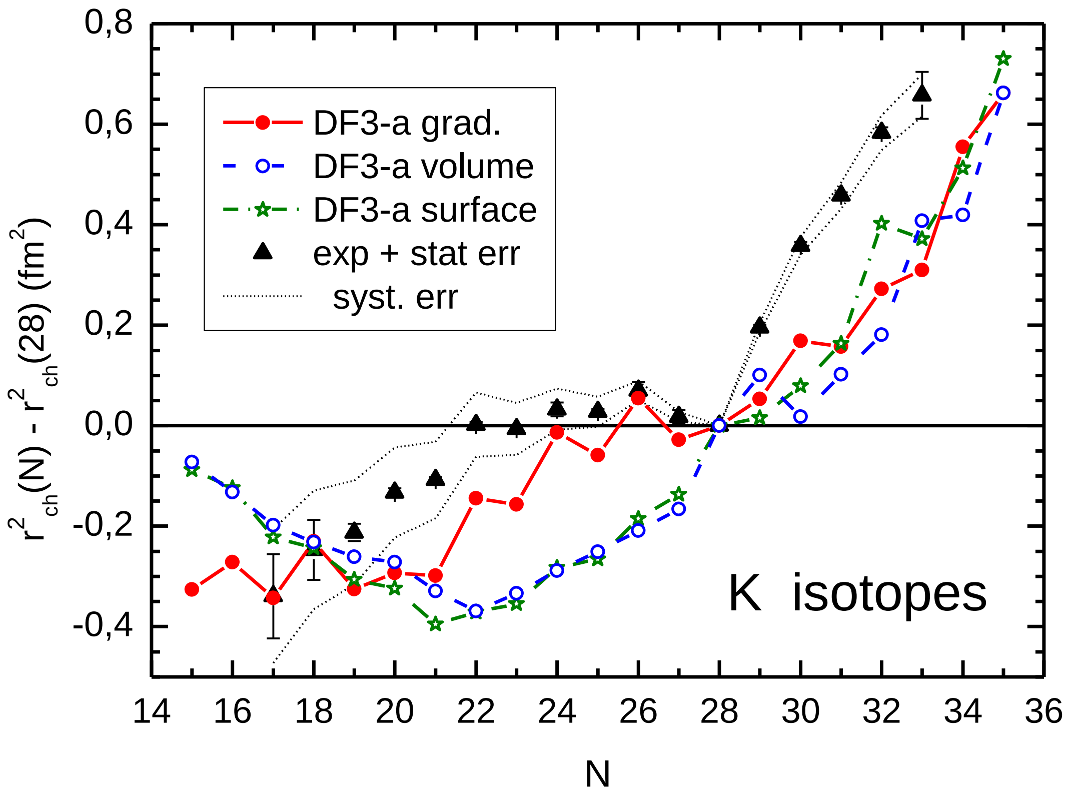


The absolute charge radii determined relative to 39K. Calculations were performed with the nuclear CC method using NNLOsat and ΔNNLOGO(450) interactions, and with the Fayans – DFT using the ***Fy(Δr, HFB)*** *energy density functional.*

Á. Koszorús et.al., ***Nature Physics****, doi.org/10.1038/s41567-020-01136-5*

Systematical (atomic physics) errors @ N~32 are rather high

Rch (rms). Form of pairing.



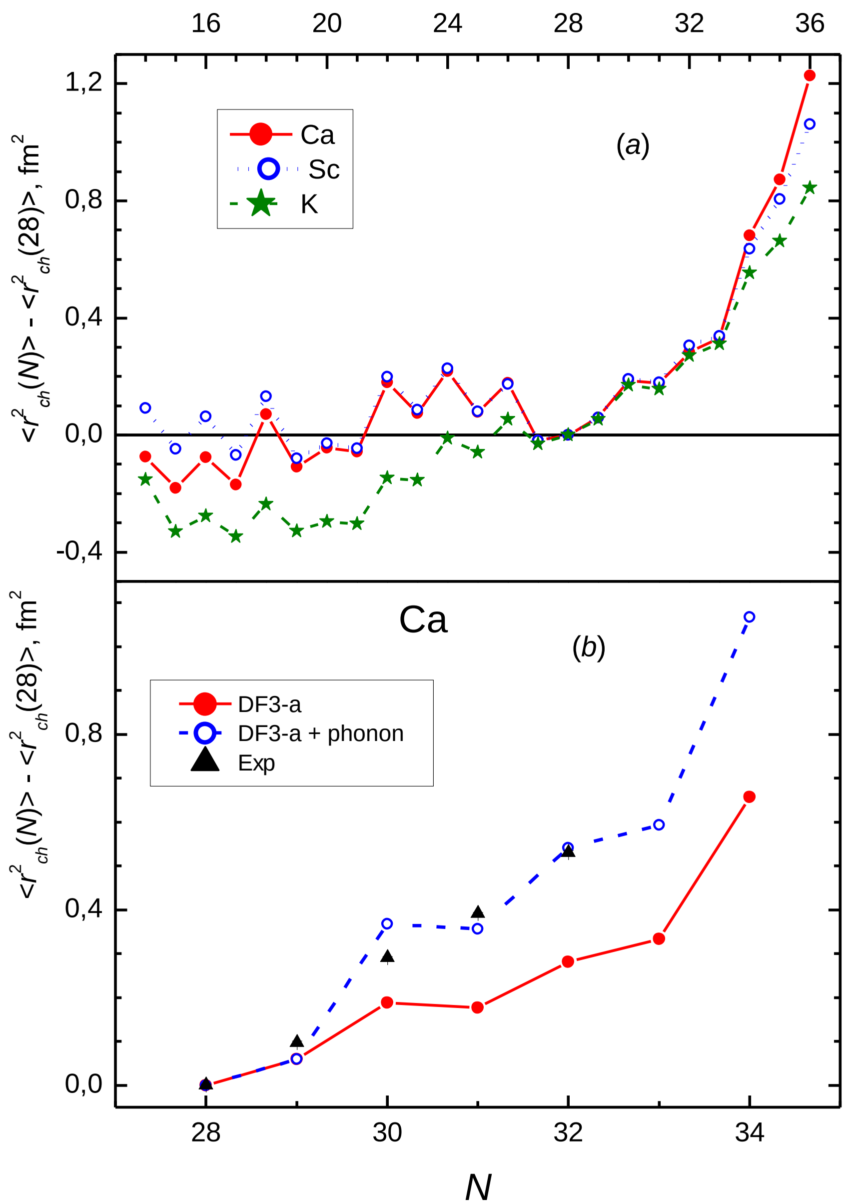
Pure volume and pure surface pairing – do not describe parabolic shape and "exaggerate" OES.

cf. U. C. Perera ,1 A. V. Afanasjev ,1 and P. Ring 2

PHYSICAL REVIEW C **104**, 064313 (2021)

Density gradient term – describes OES , smoothing OES at N>28.

Three universal features of isotopic dependence (K, Ca, Sc).



A similar N-Z dependence of dms (relative to N=28).
Easy to parametrize (up to Zn) as : ~(N-Z)
M.Kortelainen Phys.Rev . 102 (2022)

A parabolic shape (between N=20 and N=28) and OES
Described only with gradient pairing
S.A. Fayans et.al. Nucl. Phys. 49 (2000)

Universal underestimate of radii an N>28).
Non-regular (A-dependent) contribution of the quasiparticle-phonon coupling
I.N. Borzov, E.E. Saperstein, S.V. Tolokonnikov JETP Lett. 102 (2016).

Anomalous exp. charge radii in 49 -- 52 Ca isotopes

Particle-phonon contribution

(Bohr-Mottelson):

$$\delta\langle r^2\rangle_L = R_0^2 \frac{5}{4\pi}\beta_L^2,$$

Particle-phonon (FFST):

$$\delta_L\rho(\mathbf{r}) = \int \frac{d\varepsilon}{2\pi i}\delta_L G(\mathbf{r},\mathbf{r},\varepsilon),$$

$$\delta_L G = G\delta_L\Sigma G,$$

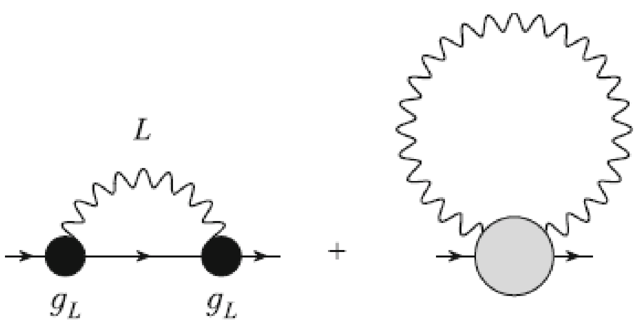


Fig. 1. Variation of the mass operator Σ in the field of an L phonon: g_L is the vortex of the production of the L phonon and the gray circle is a "tadpole" (sum of all nonpole diagrams).

DF3-a incl. 2+ and 3- phonons

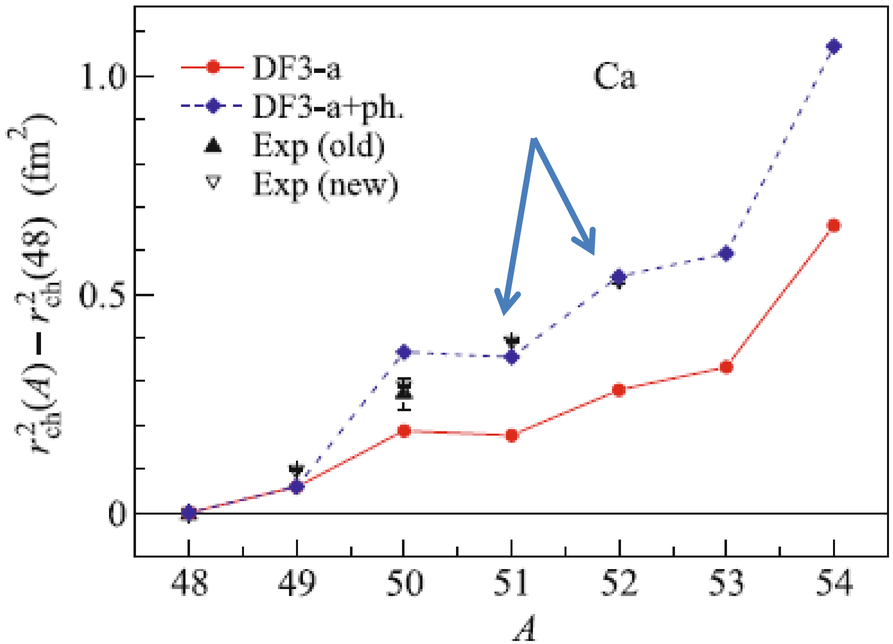
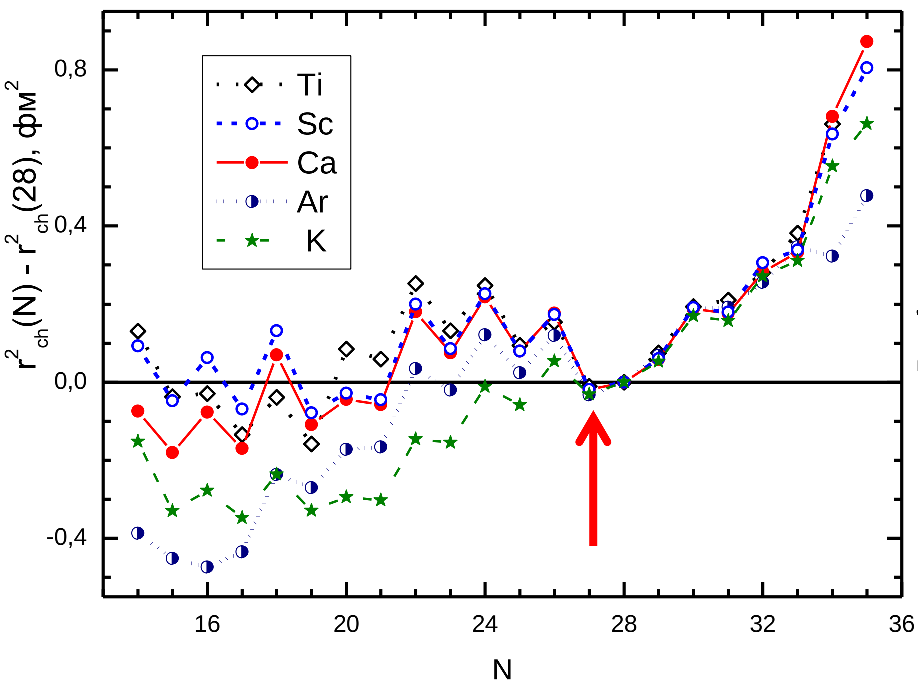
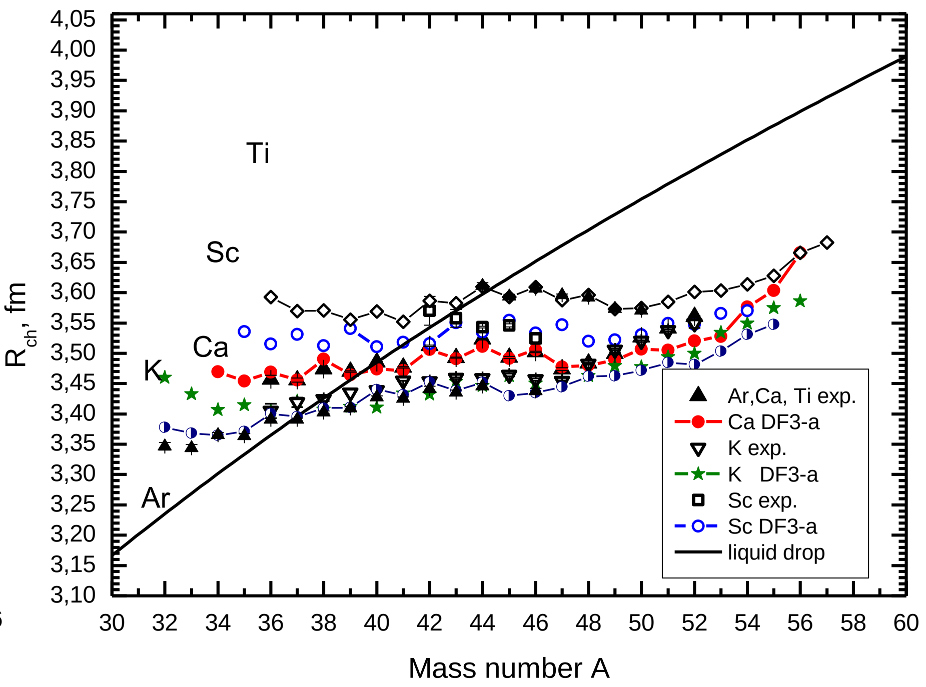


Fig. 5. (Color online) Squares of the charge radii of calcium isotopes measured from the value for ^{48}Ca.

E.E. Saperstein, I.N.Borzov, S.V. Tolokonnikov
JETP. Lett. 104 218 (2016)

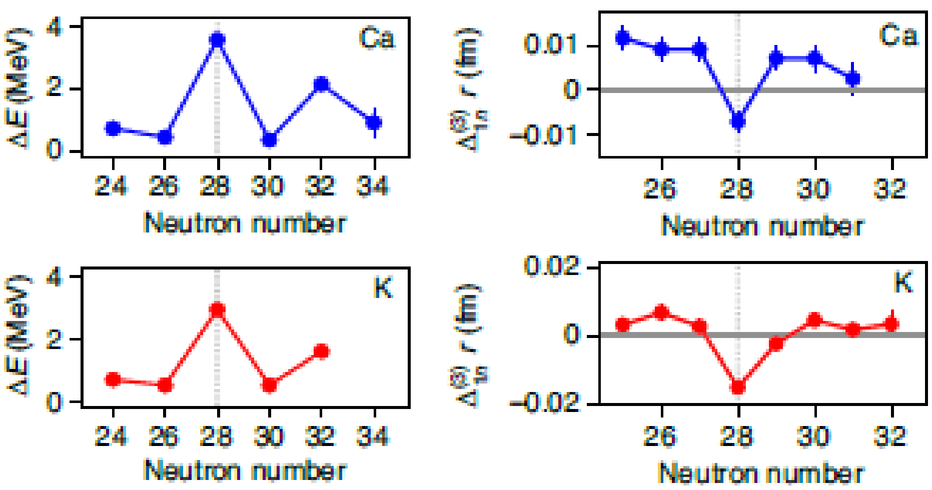
DMS charge radii Ar -Ti isotopes relative to N=28 in which the systematic uncertainties are largely cancelled out





Á. Koszorús et.al., Nat.Physics, doi.org/10.1038/s41567-020-01136-5

Charge radii of exotic potassium isotopes challenge nuclear theory and the magic character of N = 32.

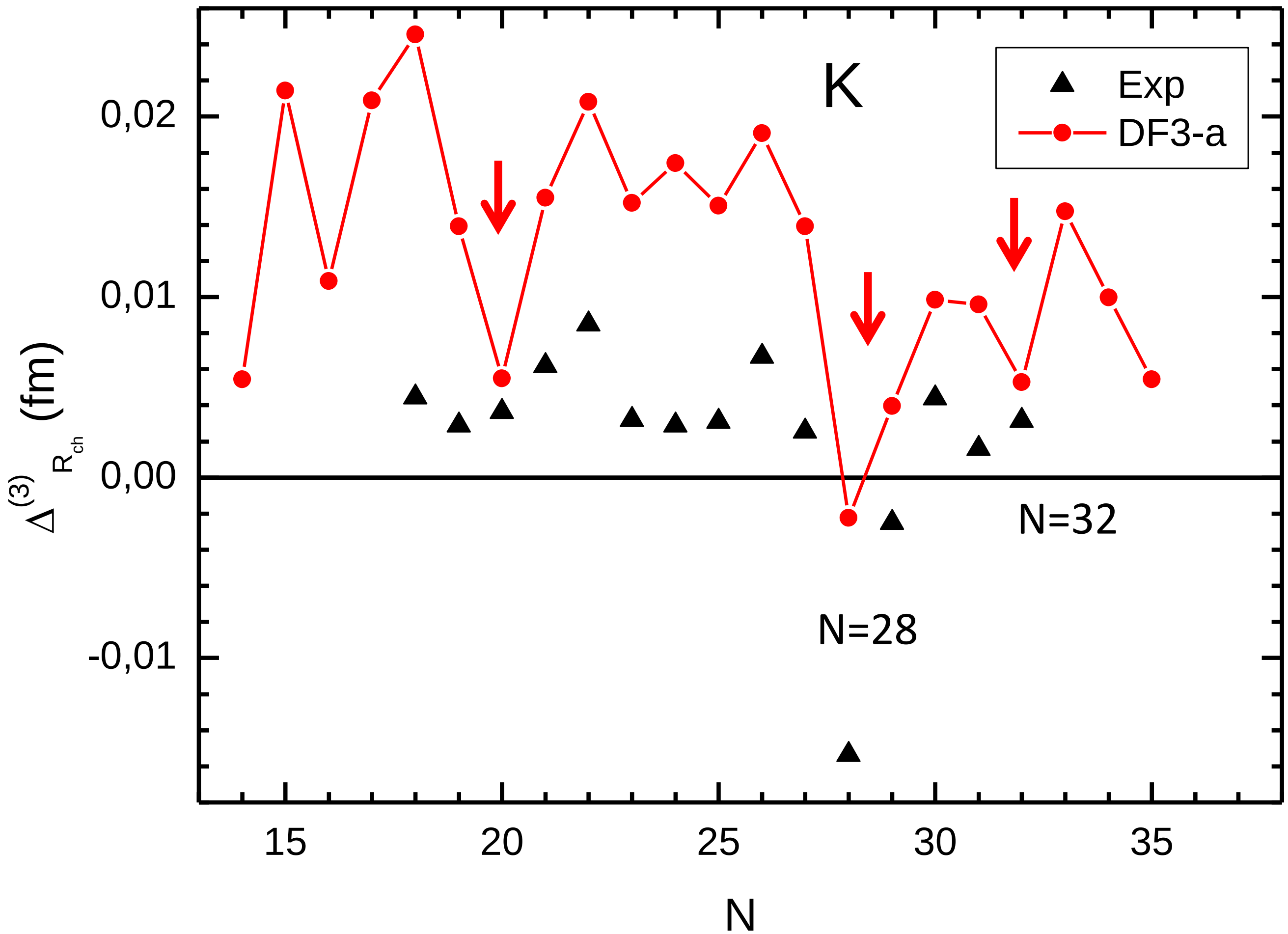


From the charge radii, the odd–even staggering (OES) can provide a signature of magicity. This is investigated by the, so-called, **"3-point filter" Δ(3)** parameter.

At well-known shell gaps, **Δ(3)** parameter is locally inverted, as shown for potassium and calcium at N = 28. However, no such inversion is seen at N = 32 for potassium.

$$\Delta(3) = 1/2\, (-1)^{N+1}\, [\, rms(N-1) - 2*rms(N) + rms(N+1)]$$

Three-point filters Δ(3) for rms-radii



All the anti-resonances are at the right places @ N=20, 28, 32.

NB!   What about the exp. @ N=20 cf. N=28

Prospects

- *So far : near-spherical nuclei with pairing (Ar,K, Ca, Sc,Ti).*
- *A challenge: deformed nuclei .*
- *The nuclear densities carry more direct info on nuclear structure than the radii. The E+M and weak densities calculations are planned with an eye on Rnp and EOS .*

Interplay of deformation,
more complex form of pairing
and phonon+qp effects.
Specific problem: odd-odd nuclei.
Yb isotopes radii anomaly– possible BSM effects

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