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*The Fayans functional.
New constraints from the equations of state.*

NUCLEAR EQUATION OF STATE

The nuclear EOS is of paramount importance for nuclear physics and nuclear astrophysics.

*At ISOSPIN ASYMMETRY PARAMETER $\delta = (\rho_p - \rho_n) / \rho = 0$ **symmetric nuclear matter (SNM)**, the EOS defines*

- the global properties of atomic nuclei and nuclear saturation density - ρ_0 ;*
- the dipole polarizability, the giant dipole resonance energy, neutron skin $R_{np} = \sqrt{\langle r_n^2 \rangle} - \sqrt{\langle r_p^2 \rangle}$.*

*At ISOSPIN ASYMMETRY $\rightarrow 1$ **pure neutron matter (PNM)**, the nuclear EOS determines the properties of neutron stars.*

THE SCOPE

Fine tuning of the Fayans functional DF3-a by varying previously unused parameter h_{-2} of the isovector volume component of EDF.

Constraints for fitting :

- a) keeping the fit quality (masses, $\epsilon_s.p$, $\langle R_{ch} \rangle \dots$)*
- b) adding condition on max. of E1 resonance 208Pb.*

The aim:

- To find an impact of h_{-2} on EOS of the SNM and PNM, as well as on $S(\rho)$, $L(\rho)$.*
- Would varying the h_{-2} allow one to meet the constraints on the EOS parameters ?*

$J=S(\rho_0)$, $L=L(\rho_0)$ and R_{np} (neutron skin)

derived from nuclear masses, PREX-II experiments, “ab initio” χ ETF calculations

and astrophysical observations: NS radii, gravitational observations: LIGO, VIRGO, NICER e.t.c.

EOS for sub-nuclear matter, nuclei, symmetric nuclear matter (SNM), pure neutron matter (PNM) :

$$E(\rho_p + \rho_n, \delta) / A$$

density dependence of total energy / per nucleon

$\rho = \rho_p + \rho_n$ - total barionic density, $\delta = (\rho_p - \rho_n) / \rho$ - isospin asymmetry

SNM ($\delta = 0$) \rightarrow PNM ($\delta = 1$)

- EOS can be constructed in empirical form (Weizsaker)
or microscopically : e.g. from Energy -Density Functional or χ EFT...*
- In micro-approach an equilibrium state of dense matter (if any)
is found selfconsistently (for each density ρ) by minimization*

$$E(\rho_0)/A \rightarrow \min \{ \varepsilon(\rho, \delta) \}$$

For uniform system few useful EOS parametrization exist.

The simplest one is a quadratic expansion on δ^2 .

Valid at $\delta \ll 1$, $\rho < 2\rho_0$, $\rho_0 = 0.164(7) \text{ fm}^{-3}$ – equilibrium density

$$E(\rho, \delta)/A = E_{SNM}(\rho, \mathbf{0})/A + S(\rho)\delta^2 + \dots$$

$$S(\rho) = \frac{1}{2} \frac{\partial^2 E(\rho, \delta)/A}{\partial \delta^2} \Big|_{\delta=0}$$

$$E_{sym}(\rho) = E(\rho, 1)/N - E(\rho, 0)/A$$

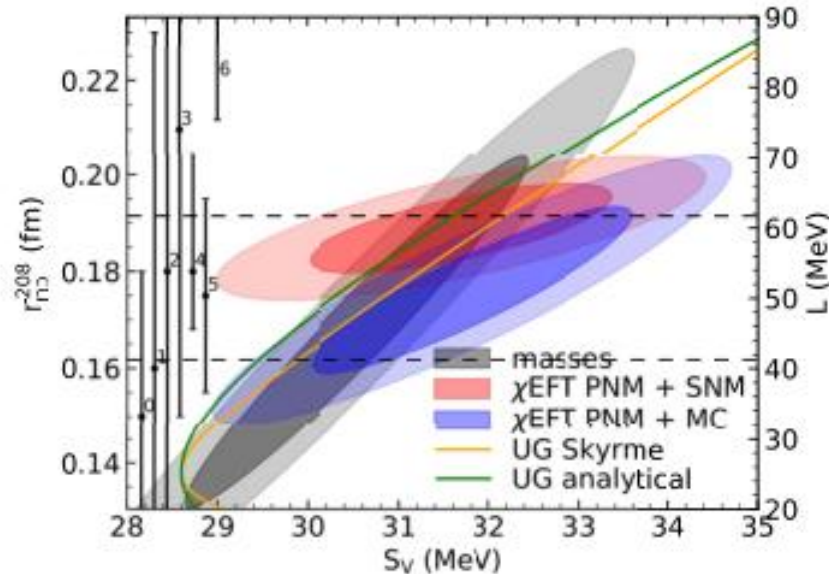
For comparing different EOS it is convenient to use the expansion parameters J, L near equilibrium density ρ_0

$J = S(\rho_0)$ – symmetry energy coefficient,

$L = 3\rho \frac{\partial}{\partial \rho} E_{sym}(\rho) \Big|_{\rho_0}$ – slope parameter $\sim \Delta R_{np}$ – neutron skin

J, L are derived from the nuclear properties and astrophysical measurements. An accuracy is still insufficient.

Constraints on $J, L, \Delta R_{np}$: masses+EFT+astro e.t.c.



$$L = 59 \pm 16 \text{ MeV}$$
$$\Delta R_{np} (208\text{Pb}) = 0.19 \pm 0.07 \text{ fm}$$

J.M. Lattimer, The Modern Physics of Compact Stars and Relativistic Gravity 2021 Yerevan, Armenia, Sep. 27 - 30, 2021

$$L = 106 \pm 37 \text{ MeV} \quad \Delta R_{np} ({}^{208}\text{Pb}) = 0.283 \pm 0.071 \text{ fm}$$

Parity violating (e, e') Jefferson Lab exp. PREX-II :

D.Adhikari et al. (PREX), Phys. Rev. Lett. 126, 172502 (2021).

Non-parametric EOS :

PREX-II, Heavy pulsar masses, LIGO/Virgo, NICER + χ EFT(2021)


$$L = 49+14-15 \text{ MeV}, \Delta R_{np}(208) = 0.17 \pm 0.004 \text{ fm}, J = 32.7+1.9-1.8$$

R.Essick, I. Tews, P. Landry, A. Schwenk Phys.Rev.Lett 127, 192701 (2021).

Volume part of the Fayans functional : DF3

$$\mathcal{E} = \frac{3}{5}\varepsilon_p(\rho_p)\rho_p + \frac{3}{5}\varepsilon_n(\rho_n)\rho_n +$$

$$+ \frac{C_0}{4} \left[a_+ \frac{1 - h_1^+(\rho/\rho_0)^\sigma}{1 + h_2^+(\rho/\rho_0)^\sigma} \rho^2 + a_- \frac{1 - h_1^-\rho/\rho_0}{1 + h_2^-\rho/\rho_0} \rho^2 \delta^2 \right].$$

$$E_{sym}(\rho) = E(\rho, 1)/N - E(\rho, 0)/A = \varepsilon_{0F} \left\{ \frac{3}{5} (2^{2/3} - 1) \left(\frac{\rho}{\rho_0} \right)^{2/3} + \frac{1}{3} a_- \frac{1 - h_1^-(\rho/\rho_0)}{1 + h_2^-(\rho/\rho_0)} \left(\frac{\rho}{\rho_0} \right) \right\}. \quad (7)$$


$a^{+,-}$, $h^{+,-}_{1,2}$, σ – iso-scalar (-vector) parameters of volume part of E/A
 Previously, $h_{2,-} = h_{2,+}$ (reducing the number of parameters)

Now, $h_{2,-}$ released to be a free parameter.

Are there $h_{2,-}$ consistent with :

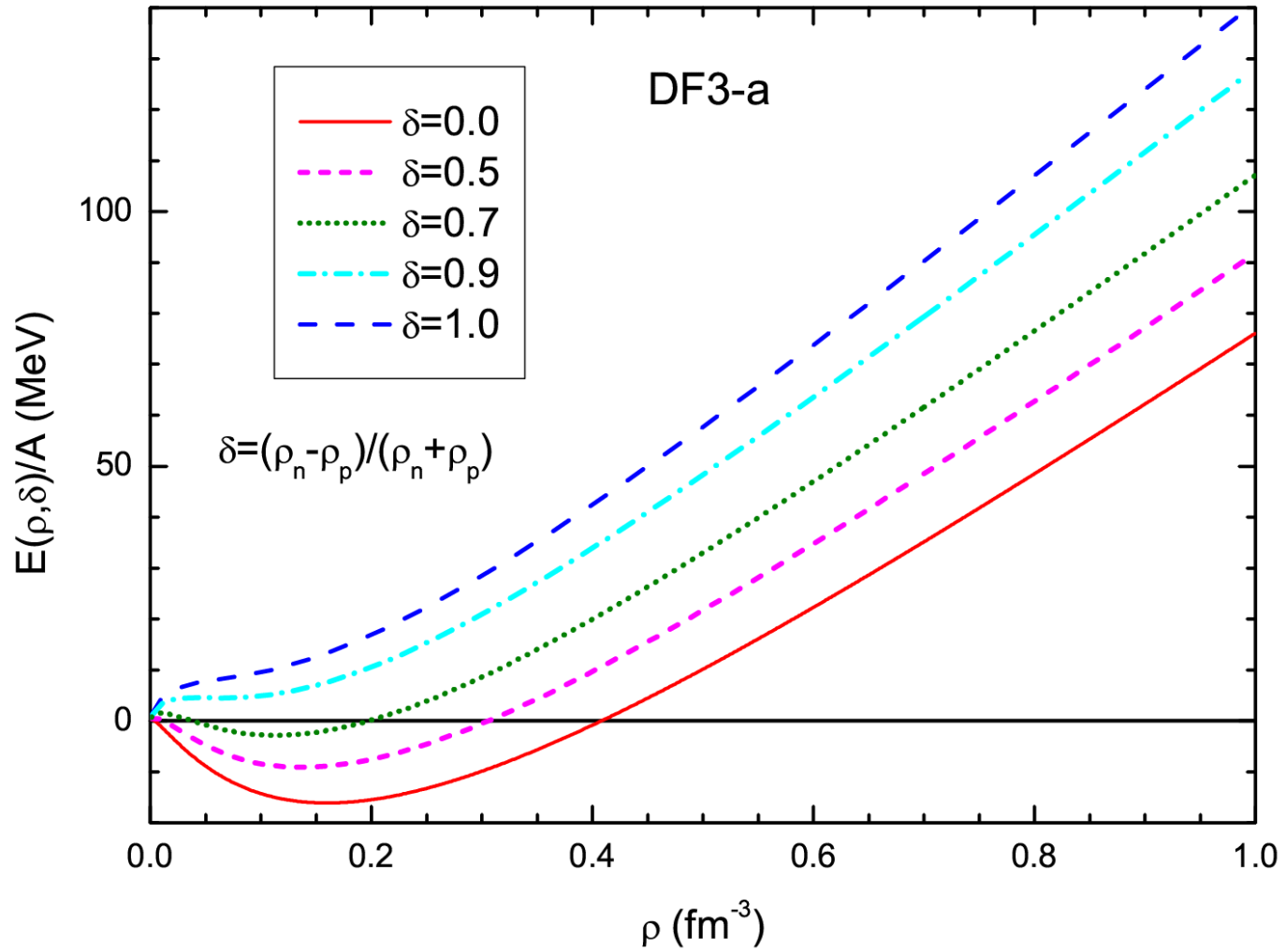
PREX-II EOS parameters:

- J , $L = 106 \pm 37$ MeV and ΔR_{np} (208Pb PREX-II) = 0.283 ± 0.071 fm ?

- with additional condition on E1 GR max energy in 208Pb

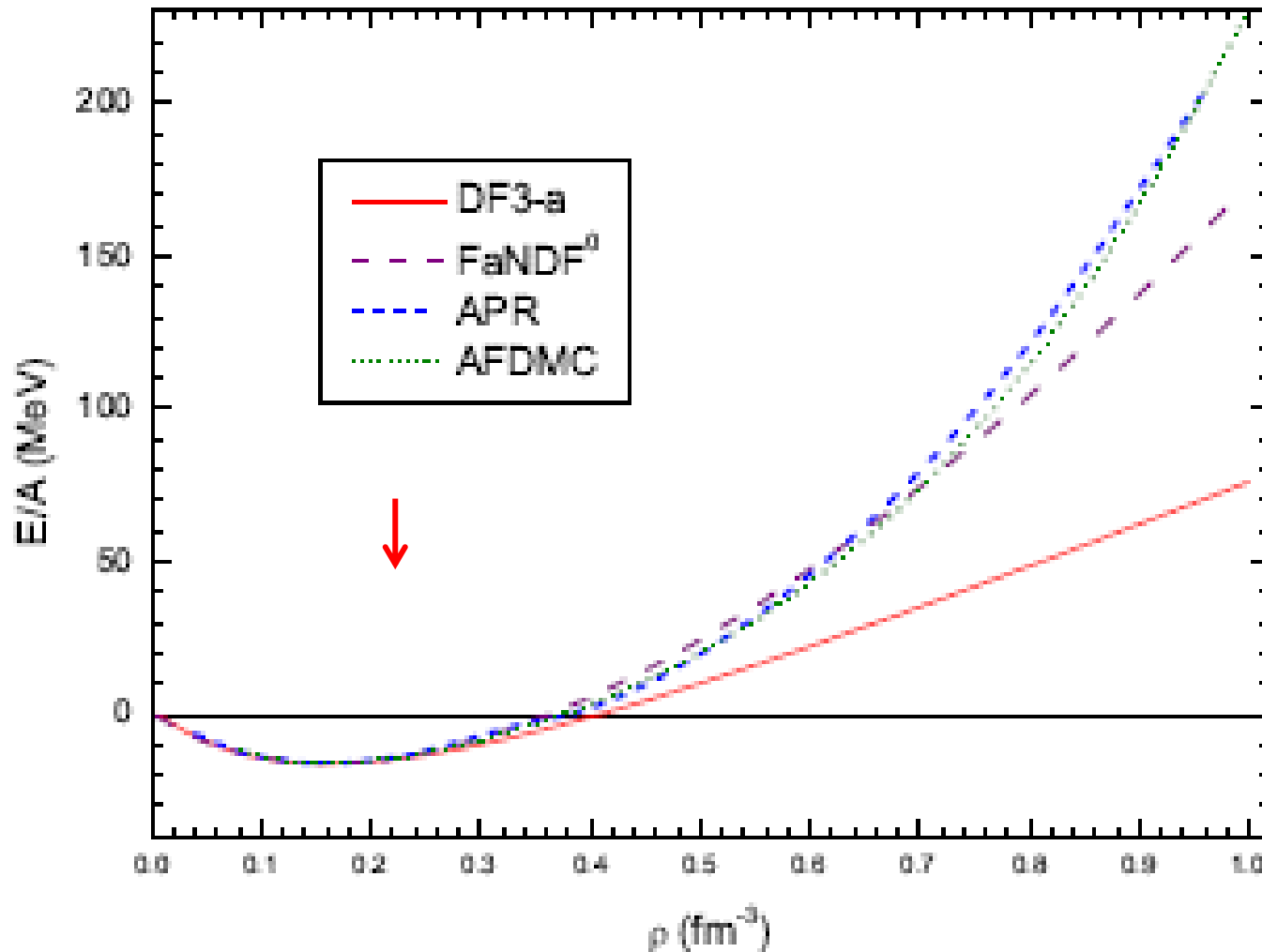
$E/A(\rho, \delta)$

Fayans functional : DF3-a



$E/A(\rho)$

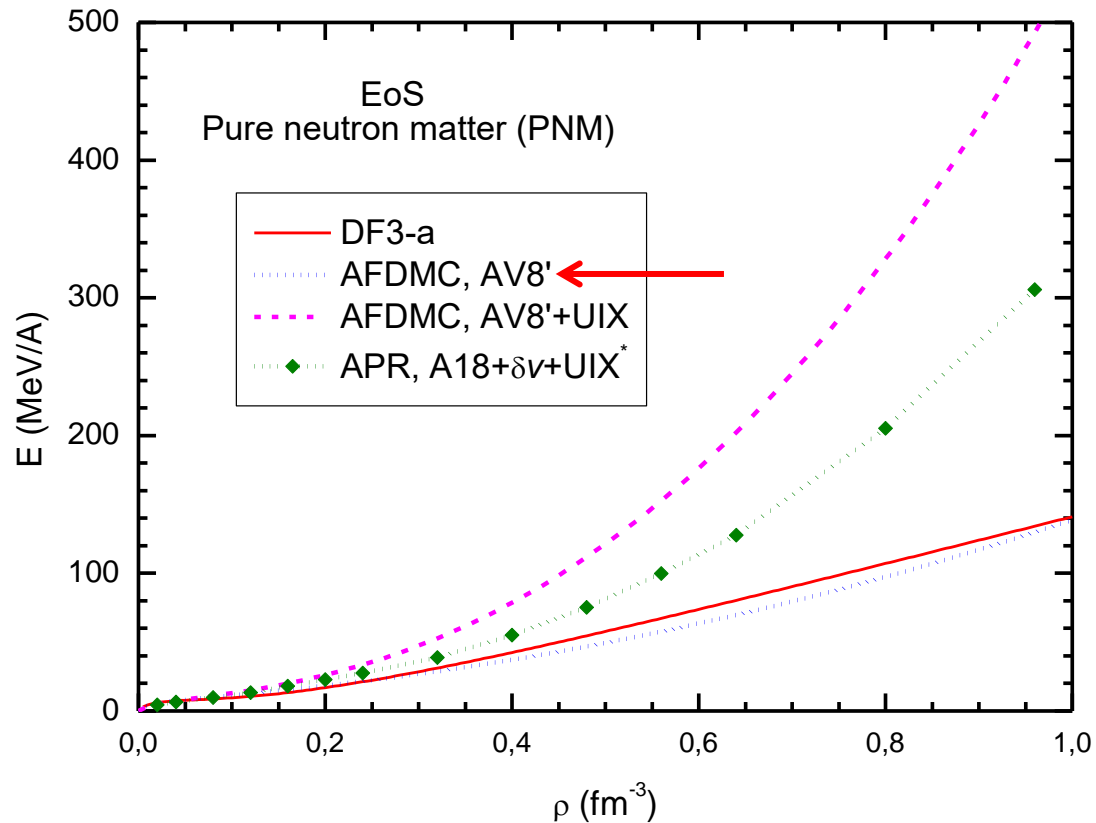
Fayans functional : DF3-a



1. Fayans S.A. *JETP Letters* 68 169 (1998)

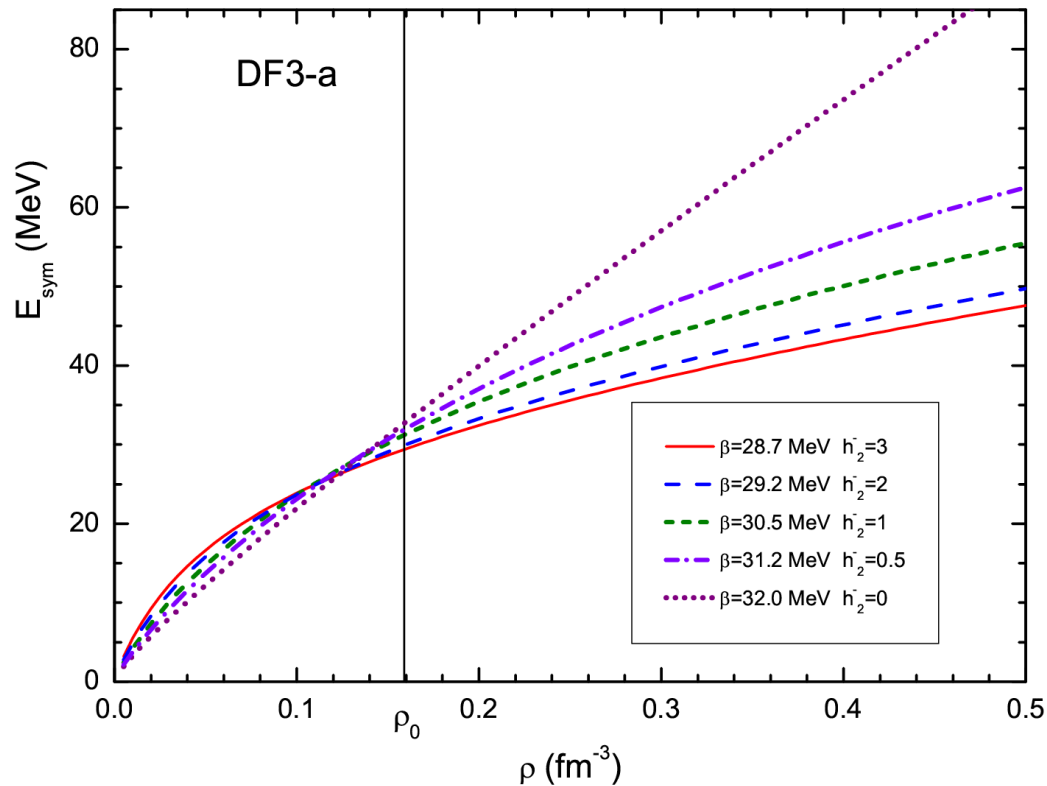
2. Akmal, V. R. Pandharipande, and D. G. Ravenhall, *Phys. Rev. C* **58**, 1804 (1998).

3. D. Lonardonì, I. Tews, S. Gandolfi, and J. Carlson, *arXiv:1912.09411 [nucl-th]* (2019).



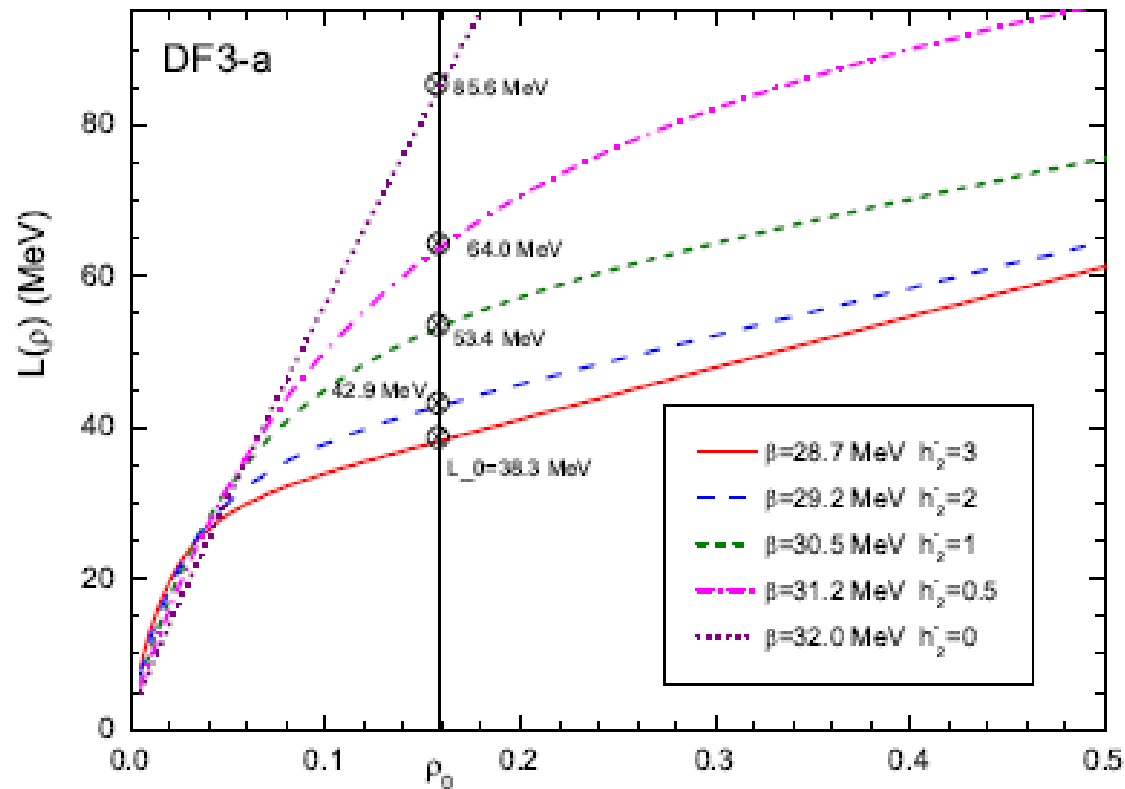
*Akmal, V. R. Pandharipande, and D. G. Ravenhall, Phys. Rev. C **58**, 1804 (1998).*
*S. Gandolfi, A. Yu. Illarionov, K. E. Schmidt, F. Pederiva, and S. Fantoni, Phys. Rev. C **79**, 054005 (2009).*

$E_{\text{sym}}(\rho, h_2^-)$ *Fayans functional : DF3-a*



DF3-a: $J=S(\rho_0) \sim 30.5$ MeV \rightarrow irrespective to h_2^-

Esym (ρ, h_2 -) *Fayans functional* : DF3-a



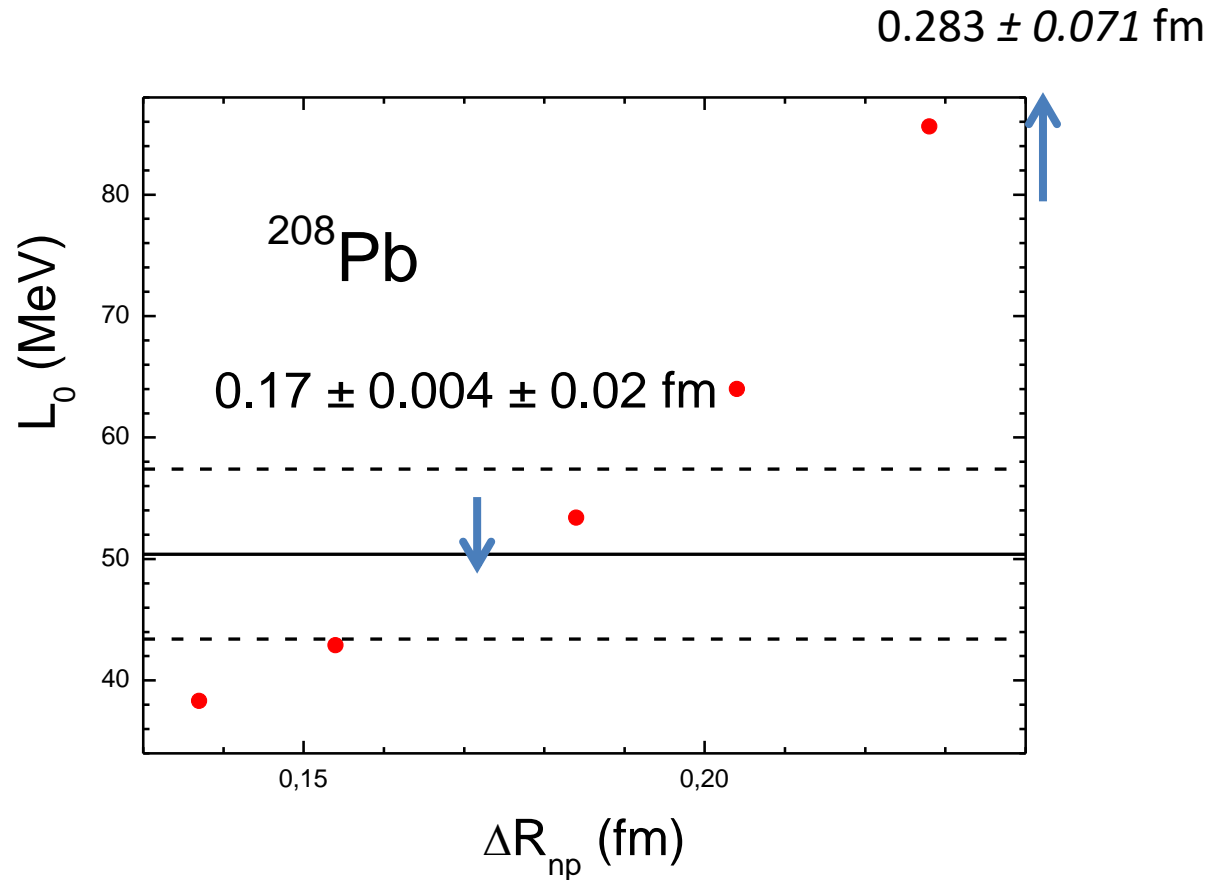
$$J = 32 \pm 3 \text{ MeV} \quad L = 59 \pm 16 \text{ MeV} \quad \rightarrow h = 0.5 - 2.0$$

$$S(\rho_0) = 30.5 \text{ MeV}, \quad L(\rho_0) = L = 49 + 14 - 15 \text{ MeV} \quad \rightarrow h > 0$$

J. Lattimer in "Nuclear Matter Symmetry Energy From Experiment, Theory and Observation", S@INT 2021.

R.Essick, I. Tews, P. Landry, A. Schwenk Phys.Rev.Lett 127, 192701 (2021)

$L(\rho, h_2^-)$ *Fayans functional : DF3-a*



$\Delta R_{np} \text{ (PREX I and II)} = 0.283 \pm 0.071 \text{ fm} - \text{no match}$

$J = 32 \pm 3 \text{ MeV} \quad L = 59 \pm 16 \text{ MeV}$

$\Delta R_{np} \text{ (PREX+Astro+}\chi\text{ETF)} = 0.17 \pm 0.004 \pm 0.02 \text{ fm}$

can be met at $2.0 > h_2^- > 0.5$

Impact of h_2^- variation on ω_{GDR} and ΔR_{np} in ^{208}Pb

Таблица I: Расчет с функционалом DF3 для различных значений параметра h_2^- .

$\omega_{GDR} = \sqrt{m_3/m_1}$, m_1, m_1 — первый и третий моменты силовой функции GDR.

h_2^-	β (MeV)	f_{in}^-	f_{ex}^-	f_{surf}^-	ω_{GDR} (^{208}Pb) (MeV)	$L(\rho_0)$ (MeV)	Δr_{np} (^{208}Pb) (fm)
0	32.0	0.808	0.808	0.808	12.80	85.6	0.228
0.5	31.2	0.775	1.163	0.969	13.37	64.0	0.204
1	30.5	0.747	1.494	1.115	13.73	53.4	0.184
2	29.2	0.694	2.080	1.387	14.11	42.9	0.154
3	28.7	0.673	2.693	1.687	14.41	38.3	0.137

$$\Delta R_{np} (\text{PREX I and II}) = 0.283 \pm 0.071 \text{ fm}$$

$$\Delta R_{np} (\text{PREX+Astro+ETF}) = 0.17 \pm 0.004 \text{ fm}$$

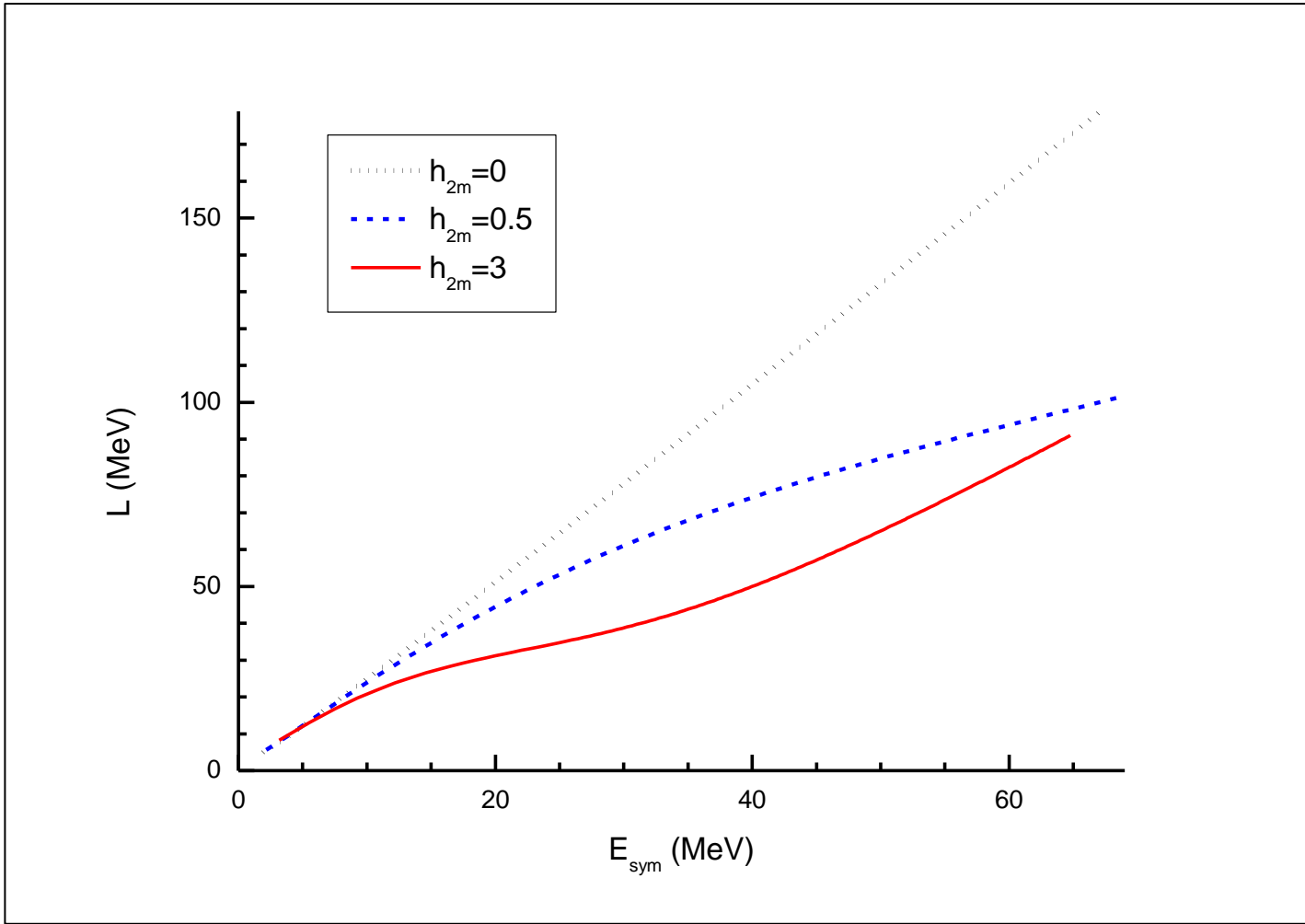
$$L = 49+14-15 \text{ MeV}$$

$$\text{as well as } J = 32 \pm 3 \text{ MeV} \quad L = 59 \pm 16 \text{ MeV}$$

are met within $h_2^- \sim 1.0 - 1.5$

Conclusions

- For the Fayans energy functional DF3-a, (previously unused) parameter of the isovector volume component h_{-2} is varied.
- Additional condition is set for the maximum energy of the giant dipole resonance. The quality of the DF 3-a fit to nuclear densities, nuclear masses, single-particle levels and charge radii has been preserved.
- The impact is studied of h_{-2} parameter on the EOS of symmetric nuclear matter (SNM), pure neutron matter (PNM), and on the density dependence of symmetry energy $S(p)$ and its derivative $L(p)$.
- The value of the neutron skin ΔR_{np} (^{208}Pb), found in the PREX I, II experiments, is not described simultaneously with the EOS SNM parameters: $J = S(p_0)$ and $L(p_0)$.
- EOS parameters $\{J, L, \Delta R_{np}\}$ by R.Essick et al. *PHYS.REV.LETT.*127, 192701 (2021) based on PREX-II + astrophysical constraints + chiral EFT calculations can be described in DF3-a within fairly narrow range of the h_{-2} parameter.



EOS for DF3 functional

$$E(\rho, \delta)/A = \mathcal{E}(\rho_p, \rho_n)/\rho,$$

$$E(\rho, \delta)/A = \varepsilon_{\text{OF}} \left\{ \frac{3}{10} \left(\frac{\rho}{\rho_0} \right)^{2/3} [(1 - \delta)^{5/3} + (1 + \delta)^{5/3}] + \frac{1}{3} a_+ \frac{1 - h_1^+(\rho/\rho_0)^\sigma}{1 + h_2^+(\rho/\rho_0)^\sigma} \left(\frac{\rho}{\rho_0} \right) + \frac{1}{3} a_- \frac{1 - h_1^-(\rho/\rho_0)}{1 + h_2^-(\rho/\rho_0)} \left(\frac{\rho}{\rho_0} \right) \delta^2 \right\}.$$

EOS : symmetric nuclear matter (SNM) - $\delta = (\rho_p - \rho_n)/\rho = 0$

$$E_{\text{SNM}}(\rho)/A = E(\rho, 0)/A = \varepsilon_{\text{OF}} \left[\frac{3}{5} \left(\frac{\rho}{\rho_0} \right)^{2/3} + \frac{1}{3} a_+ \frac{1 - h_1^+(\rho/\rho_0)^\sigma}{1 + h_2^+(\rho/\rho_0)^\sigma} \left(\frac{\rho}{\rho_0} \right) \right].$$

EOS : pure neutron matter (PNM) - $\delta = (\rho_p - \rho_n)/\rho = 1$

$$E_{\text{PNM}}(\rho)/N = E(\rho, 1)/N = \varepsilon_{\text{OF}} \left\{ \frac{3}{5} 2^{2/3} \left(\frac{\rho}{\rho_0} \right)^{2/3} + \frac{1}{3} a_+ \frac{1 - h_1^+(\rho/\rho_0)^\sigma}{1 + h_2^+(\rho/\rho_0)^\sigma} \left(\frac{\rho}{\rho_0} \right) + \frac{1}{3} a_- \frac{1 - h_1^-(\rho/\rho_0)}{1 + h_2^-(\rho/\rho_0)} \left(\frac{\rho}{\rho_0} \right) \right\}$$

One-neutron separation energies ^{208}Pb

