

**DEVELOPMENT OF A NON-EQUILIBRIUM HYDRODYNAMIC
APPROACH TO DESCRIBING THE EMISSION OF HIGH-ENERGY
SECONDARY PARTICLES IN COLLISIONS OF HEAVY IONS OF
INTERMEDIATE ENERGIES**

(FROM COLLISIONS OF SOLITONS TO DARK MATTER PRODUCTION)

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Within the framework of the non-equilibrium hydrodynamic approach, a soliton-like analytical solution of the equations for the collision of nuclear layers-slabs is found.

The prospects of the hydrodynamic approach to the description of collisions of heavy ions of intermediate energies and the importance of taking into account non-equilibrium processes are noted. When describing these spectra, the correction for the microcanonical distribution was taken into account, and the contribution of the fragmentation process was also taken into account for the proton yields .

The contribution of the effects of short-range correlations SRC, which has recently received much attention, was also studied by us. As a result, it turned out that these effects are included in our approach, since we successfully describe the experimental data on the spectra of hard photons, which are described with the addition of a high-momentum component .

Using the black body formula, it is possible to describe both the spectra of strange particles and the spectra of new particles X17 and X38 with the found temperature already for a “cold” electromagnetic plasma, when compared with experimental data for soft photons.



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Ядро-снаряд

Спектаторы снаряда

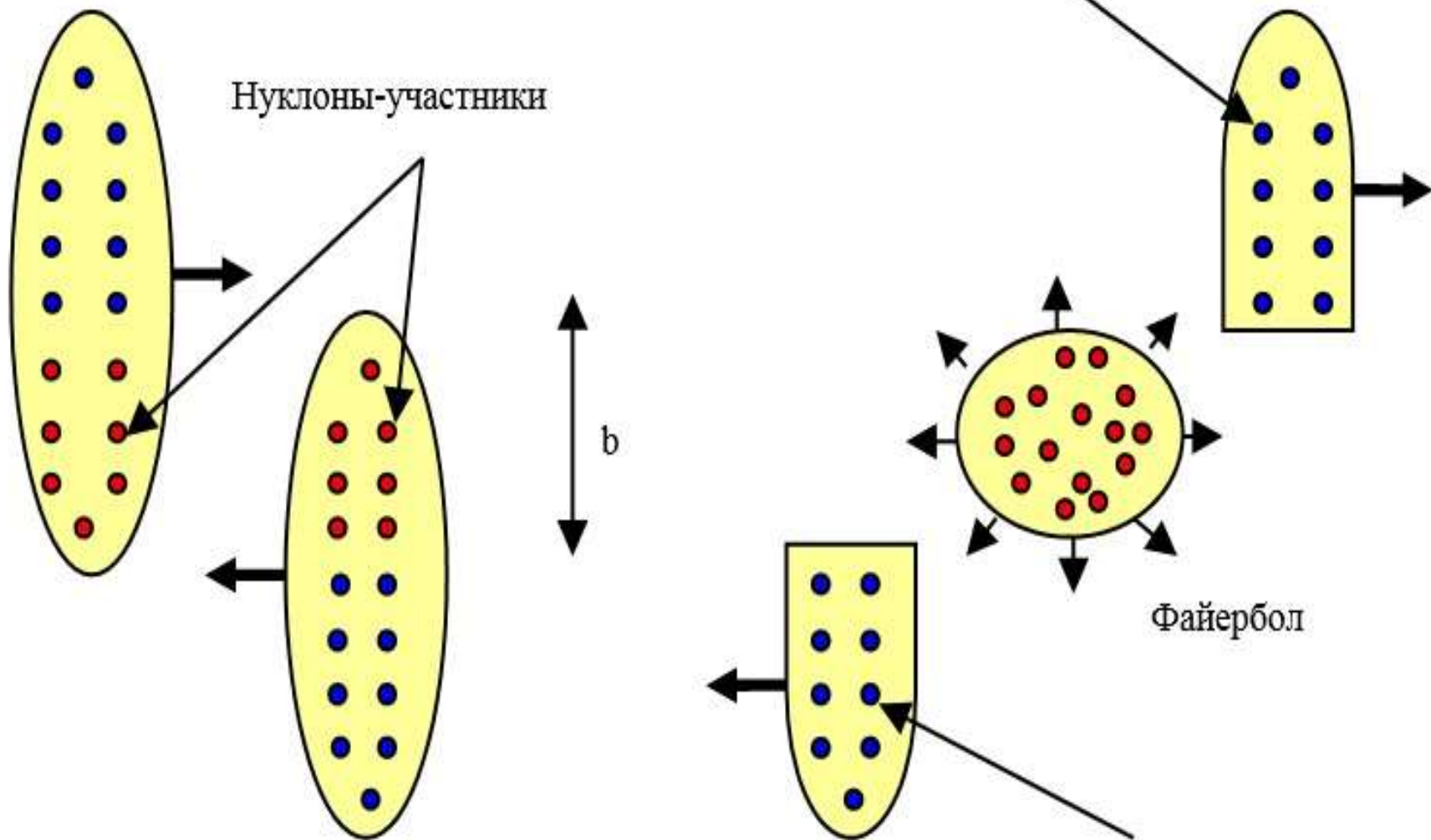
Нуклоны-участники

b

Файербол

Ядро-мишень

Спектаторы мишени



1.KORTEVEG-DE VRIES SOLITONS

$$\frac{\partial \rho}{\partial t} + v \frac{\partial \rho}{\partial x} + \rho \frac{\partial v}{\partial x} = 0 \quad P(\rho) = a\rho + b\rho^2 + \alpha \frac{d^2 \rho}{dx^2} \quad a\rho_0 + b\rho_0^2 = 0 \quad I = I_1 \left(\frac{\rho}{\rho_0} \right)^3$$

$$\frac{\partial v}{\partial t} + v \frac{\partial v}{\partial x} + \frac{1}{m\rho} \frac{\partial P}{\partial x} = 0 \quad \rho v'(\rho) = \pm \sqrt{\frac{\partial P}{m \partial \rho}} \approx \pm \left[c_{so} + \beta(\rho - \rho_0) + \frac{\alpha}{2mc_{so}} \frac{\partial}{\partial \rho} \left(\frac{\partial^2 \rho}{\partial x^2} \right) \right] = \pm c_s(\rho)$$

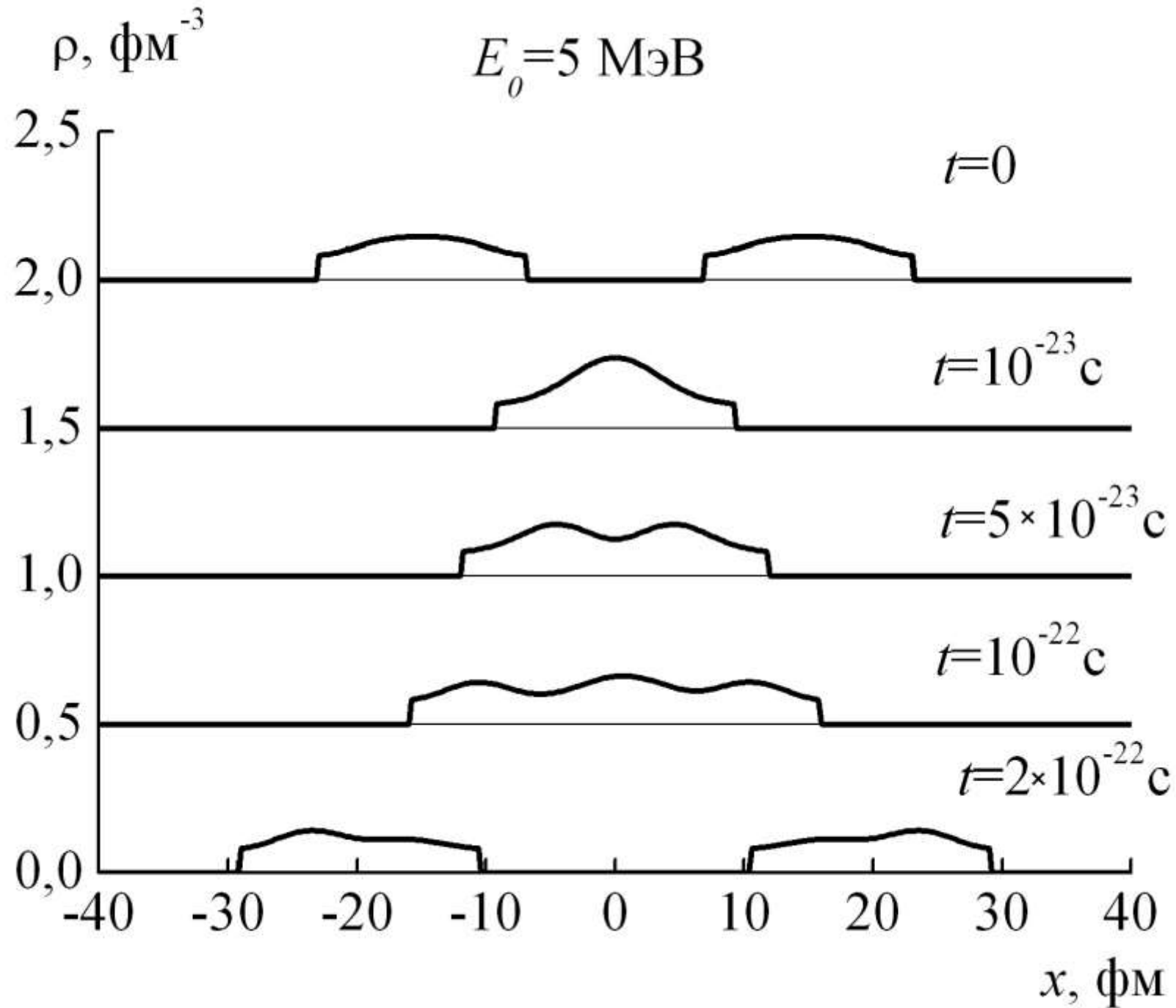
$$\frac{\partial I}{\partial t} + v \frac{\partial I}{\partial x} + 3I \frac{\partial v}{\partial x} = 0 \quad v = \pm \int_{\rho_0}^{\rho} \frac{c_s(\rho)}{\rho} d\rho + v_0 \quad c_{s0} = \sqrt{\frac{a + 2b\rho_0}{m}} \approx 1/3c \approx 10^8 \tilde{n}$$

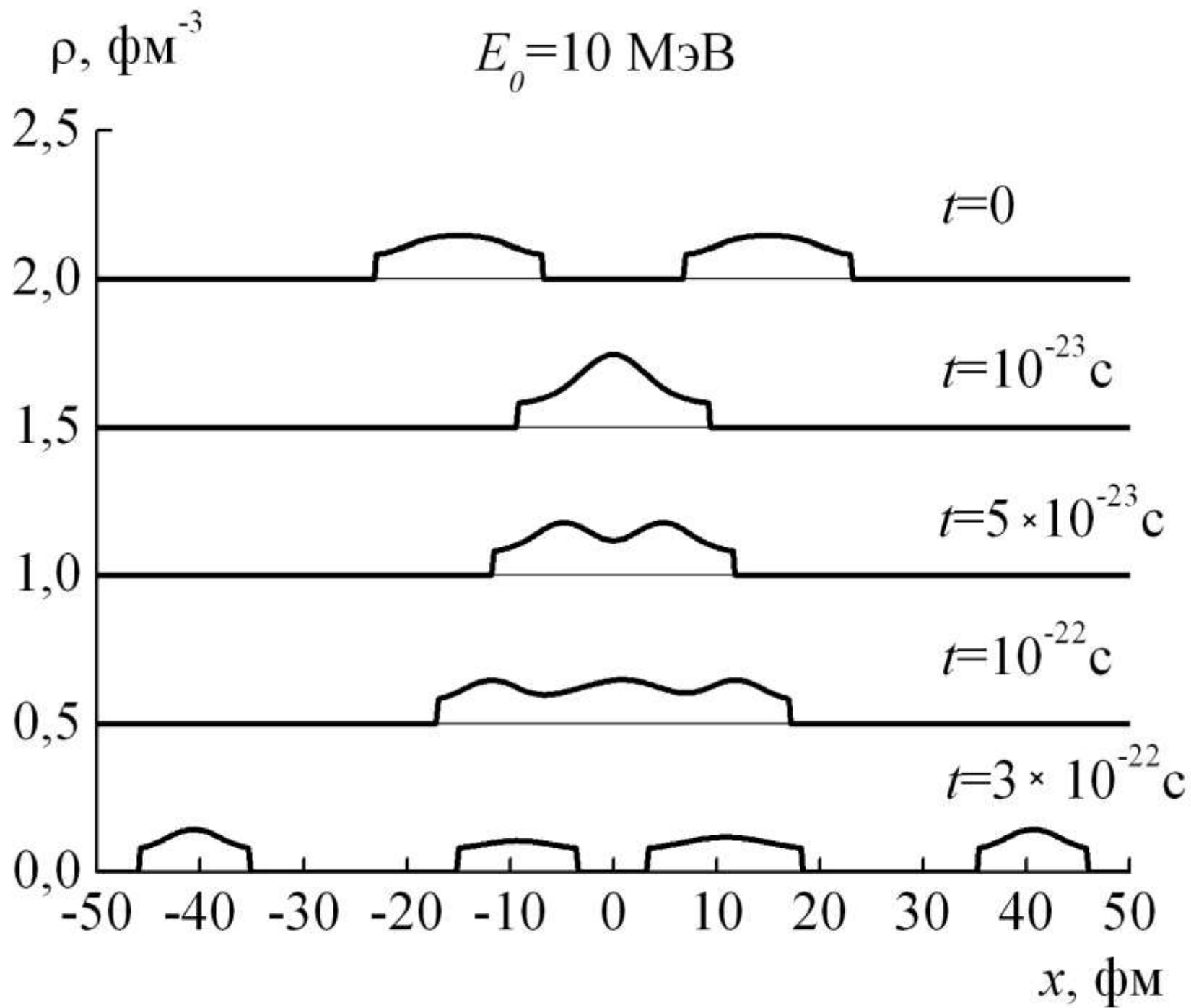
$$\frac{\partial \zeta}{\partial t'} + 6\zeta \frac{\partial \zeta}{\partial x'} + \frac{\partial^3 \zeta}{\partial x'^3} = 0 \quad \zeta = \left[\pm v_0 + c_{so} + 2c_{so} \frac{(\rho - \rho_0)}{\rho_0} \right] \frac{1}{A} \quad \zeta = 2 \frac{\partial^2 \ln s}{\partial x'^2}$$

$$s = 1 + \exp(\omega t' + k(x' - x_1))$$

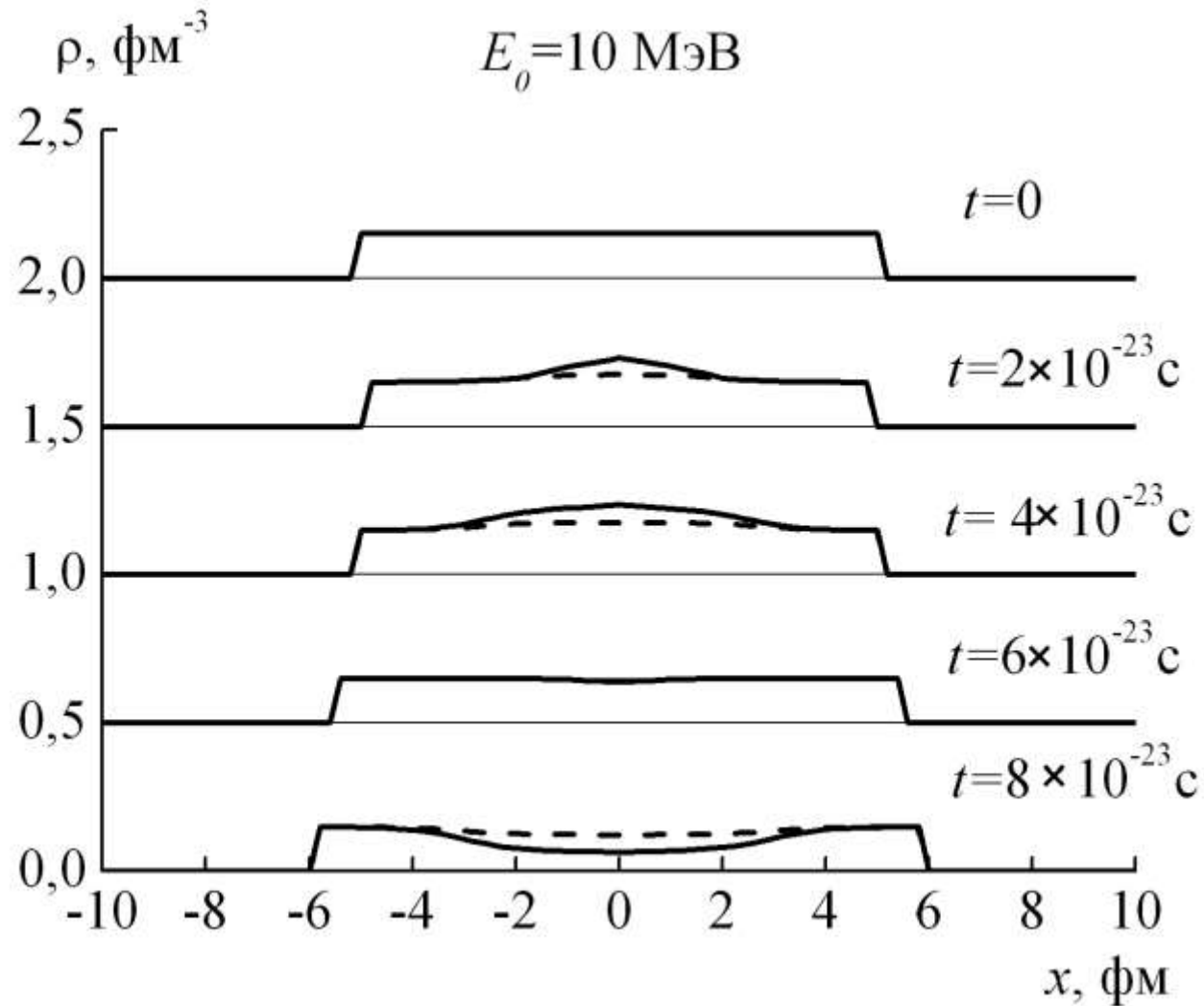
$$\omega = -k^3$$

$$Z = \int_0^L \xi \frac{dx_1}{L}$$

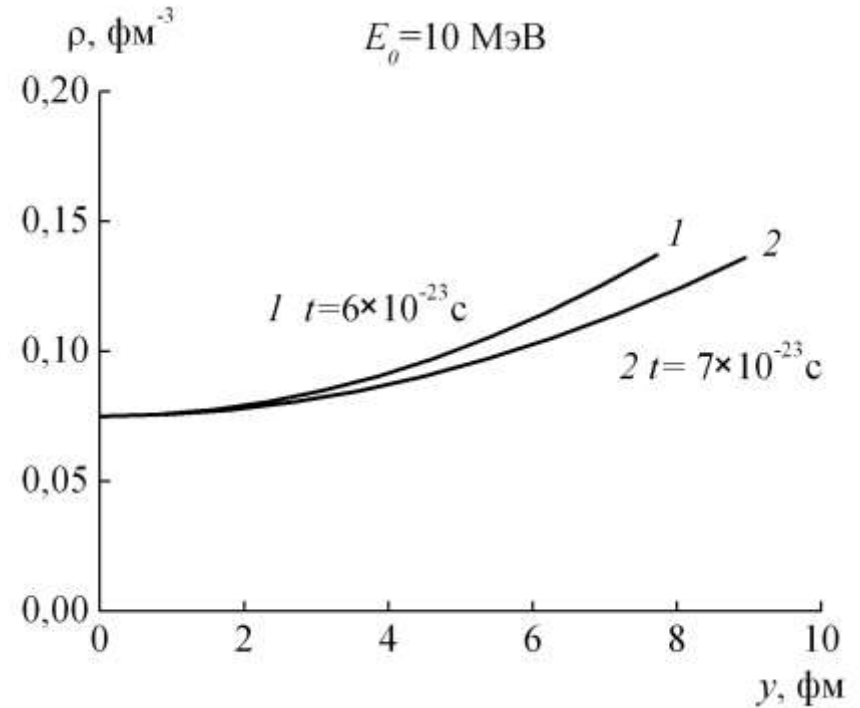
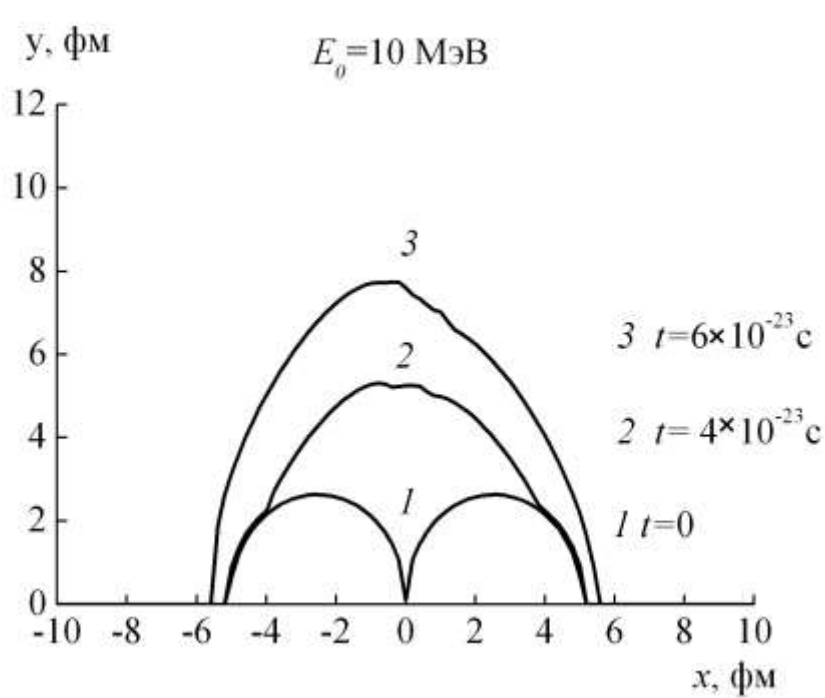




Shock wave formation



Formation of bubble



$$D = -\frac{\rho_0 v_0}{\rho - \rho_0} \quad P = -\frac{\partial(e/\rho)}{\partial(1/\rho)} = K(\rho^2 - \rho_0^2) - \alpha \left(\frac{\partial \rho}{\partial x} \right)^2$$

$$\rho = \rho_0 + 4 \frac{(\rho_1 - \rho_0)}{(\exp(-\lambda x/2) + \exp(\lambda x/2))^2}$$

$$c_s = \sqrt{\frac{\partial P}{m \partial \rho}} = D$$

$$2K\rho_1 = \frac{(\rho_0 v_0)^2}{(\rho_1 - \rho_0)^2}$$

J. Decharge, J.-F. Beger, K. Dietrich, and M.S. Weiss, Phys. Lett. B 451
275 (1999). **Z>120**

2. FEATURES OF NON-EQUALIBRIUM HYDRODYNAMIC APPROACH

- To describe the collisions of heavy ions we use the non-equilibrium hydrodynamic approach, in which the kinetic equation for the nucleon distribution function is solved jointly with the equations of hydrodynamics, which are essentially local laws of conservation of mass, momentum and energy.

$$\frac{df}{dt} = -\frac{f_0 - f}{\tau}, \quad (1)$$

- where $f_0(\mathbf{r}, \mathbf{p}, t)$ is the locally equilibrium distribution function and
- τ is the relaxation time

$$W(\rho) = \alpha\rho + \beta\rho^\chi, \quad (2)$$

- The self-consistent potential appearing in the interaction term is specified in just the same way as this is done in the case of density-dependent forces belonging to the Skyrme type

- $\rho = \int \frac{fd^3\vec{p}}{(2\pi\hbar)^3} \quad \tau = \lambda / v_T \quad W(\rho) = \alpha\rho + \beta\rho^\gamma,$
- $f(\vec{r}, \vec{p}, t) = f_1 \cdot q + f_0 \cdot (1 - q)$

(3)

- $P_{kin}^{\parallel} = P_{(kin)11} = 2(\varepsilon_1 + I_1)q + \frac{2}{3}(\varepsilon + I)(1 - q) \quad \varepsilon_1 = \frac{\hbar^2}{10m} \left(\frac{3}{2} \pi^2 \rho_0 \right)^{2/3} \frac{\rho^3}{\rho_0^2}$

- $P_{kin}^{\perp} = P_{(kin)22} = P_{(kin)33} = 2\varepsilon_2 + \frac{2}{3}(\varepsilon + I)(1 - q) \quad \varepsilon_2 = \frac{\hbar^2}{10m} \left(\frac{3}{2} \pi^2 \rho_0 \right)^{2/3} \rho$

- $\varepsilon = \frac{3}{10} \frac{\hbar^2}{m} \left(\frac{3}{2} \pi^2 \rho \right)^{2/3} \cdot \rho \quad I = \int \frac{p^2}{2m} \delta f \frac{d^3\vec{p}}{(2\pi\hbar)^3} \quad I_1 = \int \frac{p^2}{2m} \delta f_1 \frac{d^3\vec{p}}{(2\pi\hbar)^3}$

- $P_{ij} = P_{(kin)ij} + P_{int} \delta_{ij} \quad e = \varepsilon + I + e_{int} \quad e_{int} = \int_0^\rho W(\rho) d\rho \quad P_{int} = \rho^2 \frac{d(e_{int} / \rho)}{d\rho}$
- $\frac{\partial}{\partial t} ((\varepsilon_1 - \varepsilon_2 + I_1)q) + \frac{\partial}{\partial x_1} (v_1(3\varepsilon_1 - \varepsilon_2 + 3I_1)q) + \sum_{i=2,3} \frac{\partial}{\partial x_i} (v_i(\varepsilon_1 - \varepsilon_2 + I_1)q) +$

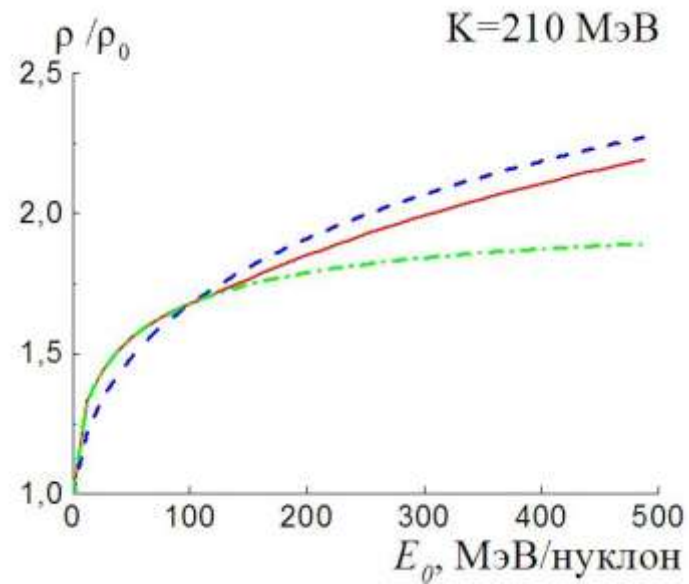
- $\rho v_1 \frac{\partial W}{\partial x_1} - \sum_{i=2,3} \frac{\rho v_i}{2} \frac{\partial W}{\partial x_i} = - \frac{(\varepsilon_1 - \varepsilon_2 + I_1)q}{\tau}$

(4)

$$p_1^2 - (p_2^2 + p_3^2) / 2$$

3.THE HYDRODYNAMIC STAGE

- After selecting the region of the local heating, **hot spot - the overlap region of the colliding nuclei**, we analyze the stages of compression, expansion and freeze-out of matter during the collision of heavy ions. At the compression stage, **a collisionless shock wave with a changing front** is formed. At the expansion stage, when the shock wave reaches the boundaries of hot spot, the initially compressed system is expanded, we describe it in the relaxation approximation taking into account the **nuclear viscosity**. As the relaxation time we take $\tau = \lambda / v_T$ where $\lambda = 1/\sigma\rho$ is the mean free path, $\sigma \approx 40\text{mb}$ is the total nucleon-nucleon cross section, ρ is the nucleon density, and
- v_T is the average velocity of the thermal motion of the nucleons. At the freeze-out stage, when the system reaches a critical density also called the freeze-out density, the system does not "hold itself" and the **secondary particles** are formed.



- Fig. 1. Dependence on the collision energy of the maximum compression ratio
- ρ/ρ_0 achieved in the central collision of nuclei for the case of the relaxation factor
- q (solid line) calculated by us, for the case when the factor $q=0$
- (dashed line), and for the case when $q=1$ (a dashed-dot line)

4. STATISTICAL FRAGMENTATION MECHANISM

- To describe the soft part of the spectrum of emitted protons, one can use the statistical model of fragmentation of colliding heavy ions proposed by **Feshbach, Huang, and Goldhaber**. According to this model, the probability of the escape of fragments from a compound nucleus is proportional to

$$\exp\left(-\frac{p^2}{2\sigma_K^2}\right)$$

- where p is the momentum of the fragment in the rest frame of the nucleus, and the variance

- $$\sigma_K^2 = \sigma_0^2 \frac{K(A-K)}{A-1} \quad 1) \quad \sigma_0^2 = \frac{\langle p^2 \rangle}{3} = \frac{1}{3} \frac{3}{5} p_F^2, \quad p_F = \sqrt{2m(E^* - 3/2T)} \quad 2) \quad \sigma_0^2 = mT$$

- As a result, we find the contribution we need to the cross section for **protons during fragmentation** (b is the impact parameter)

$$E \frac{d^2\sigma}{p^2 dp d\Omega} = \frac{2\pi}{(2\pi\hbar)^3} \int b db \int C d\mathbf{r} r \gamma(E - \vec{p}\vec{v}) \exp\left(-\frac{(\mathbf{p} - \mathbf{p}_0)^2}{2\sigma_0^2}\right)$$

5. A COMPARISON WITH EXPERIMENTAL DATA

- As a result, the double differential cross-section of proton emission has the form (where b is the impact parameter, \hbar is a Planck constant, \vec{r} is the radius vector):

$$E \frac{d^2\sigma}{p^2 dp d\Omega} = \frac{2\pi}{(2\pi\hbar)^3} \int G(b) b db d\vec{r} \gamma(E - \vec{p}\vec{v}) f(\vec{r}, \vec{p}, t) \quad (5)$$

- where the distribution function of emitted protons

$$f(\vec{r}, \vec{p}, t) = g \left[\exp\left(\frac{\gamma(E - \vec{p}\vec{v} - \mu) + T\delta}{T}\right) + 1 \right]^{-1} \quad (6)$$

- Here the spin factor $g = 2$, $E = \sqrt{p^2 + m^2}$, γ and \vec{p} are respectively the total energy, the Lorentz factor and the proton momentum; $\vec{v}(\vec{r}, t)$ is the velocity field, $G(b)$ is the factor taking into account that the cross section of the hot spot formation is always greater than the geometric one, μ is the chemical potential, which is found from the conservation of the average number of particles for a grand canonical ensemble, T is the temperature, δ is correction for the microcanonical distribution, which for the kinetic energy $\varepsilon = E - m > E_1$ is equal to

$$\delta = \left[-M \ln \left(1 - \frac{\gamma(E - \vec{p}\vec{v}) - m}{MT} \right) - \frac{\gamma(E - \vec{p}\vec{v}) - m}{T} \right] \quad (7)$$

where $M = 3N / 2$, N is the number of nucleons in the thermostat, $E_1 (E_1 \gg T)$ is the energy that is close to the energy of the thermostat, i.e. close to the kinematic limit for the energy of the system. We also chose the energy value $E_2 (E_2 < E_1)$, when the distribution function decreases by an order of magnitude compared to its maximum. When $\varepsilon < E_2$ the amendment δ was supposed equal to zero. In the energy interval $E_2 > \varepsilon > E_1$ it was a linear interpolation between zero and expression (7). Here the correction δ is found for the Boltzmann limit of an ideal gas, since deviations from a grand canonical distribution of the Fermi gas are manifested on the "tails" of the energy spectra when the Fermi distribution coincides with the Boltzmann limit.

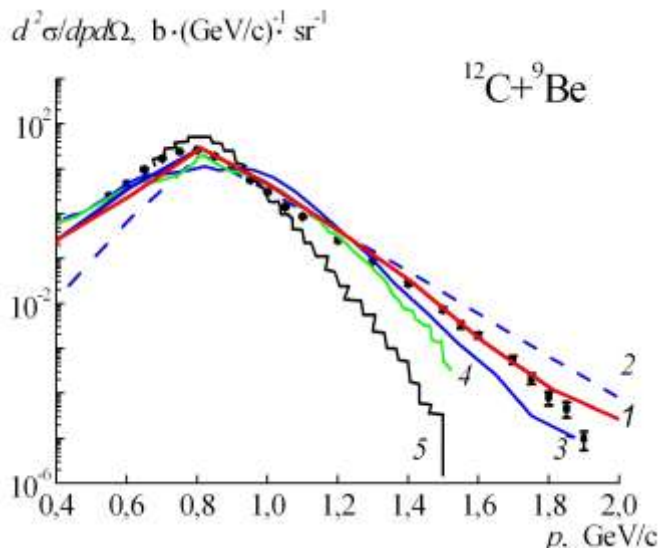
- The probability of a microcanonical distribution in the limit of the Boltzmann limit of an ideal gas is

- $$w(\vec{r}, \vec{p}) = C_M \left(1 - \frac{\varepsilon}{U} \right)^M = C_M \exp \left(M \ln \left(1 - \frac{\varepsilon}{MT} \right) \right) \quad , \quad (8)$$

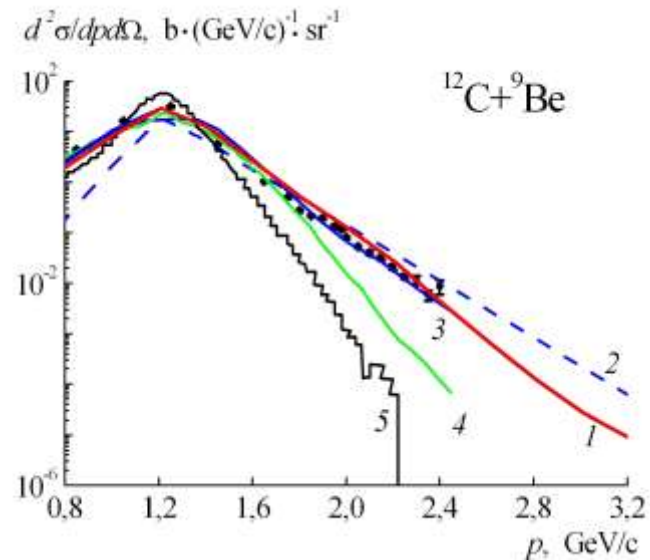
- where ε is the kinetic energy of the system, $U = MT$ is the energy of the thermostat, C_M is the normalization factor. As a result, in the limit of a large number of particles N at $M = \frac{3}{2}N \rightarrow \infty$, expression (8) becomes a grand canonical distribution

- $$w_0(\vec{r}, \vec{p}) = C_M \exp\left(-\frac{\varepsilon}{T}\right)$$

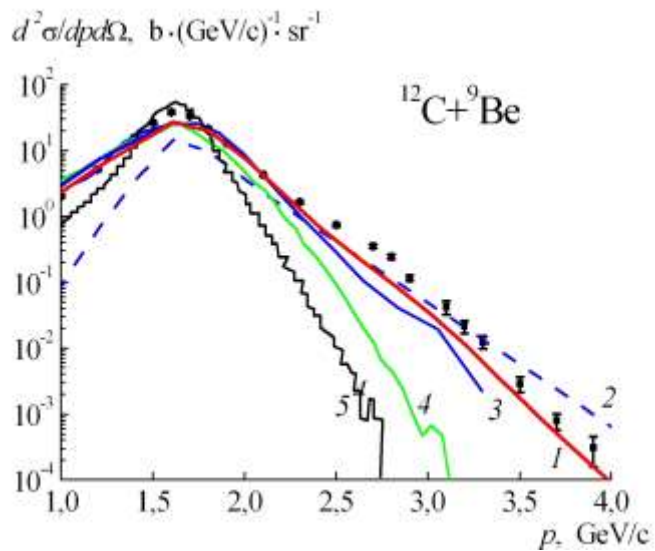
- Thus, on the tails of the energy distributions, using formula (7), we find an amendment for the microcanonical distribution (6), which changes the usual Fermi-Dirac distribution, describing the system well away from the tails of the proton spectrum. Moreover, in formulas (5) - (6) it is taken into account that the energy of the system is recalculated in accordance with the Lorentz transformations. The energy in the distribution (6) is reckoned from the value of the self-consistent mean field **with allowance for the surface energy**, since the nucleons are "locked" by the mean field.
- In addition to the contribution of (1) to the cross section for the emission of protons from the hot spot, we also took into account the contribution from the fusion of the non-overlapping parts of the colliding nuclei **so called "spectators"**.



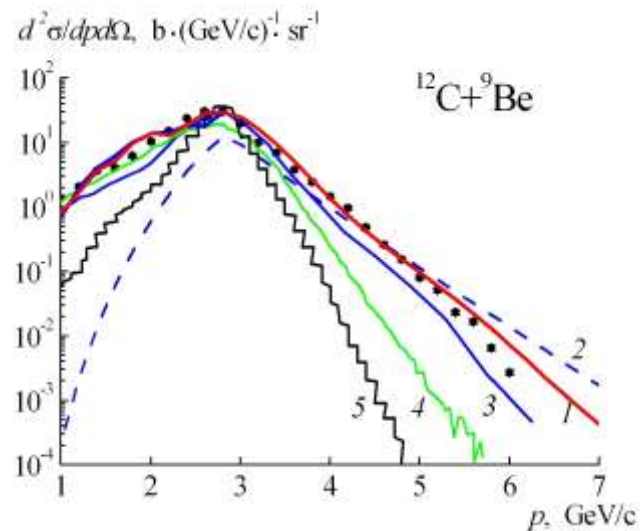
$^{12}\text{C}+^9\text{Be}$ 300 MeV/nucleon.
(protons) $3,5^0$



$^{12}\text{C}+^9\text{Be}$ 600 MeV/nucleon.
(protons) $3,5^0$

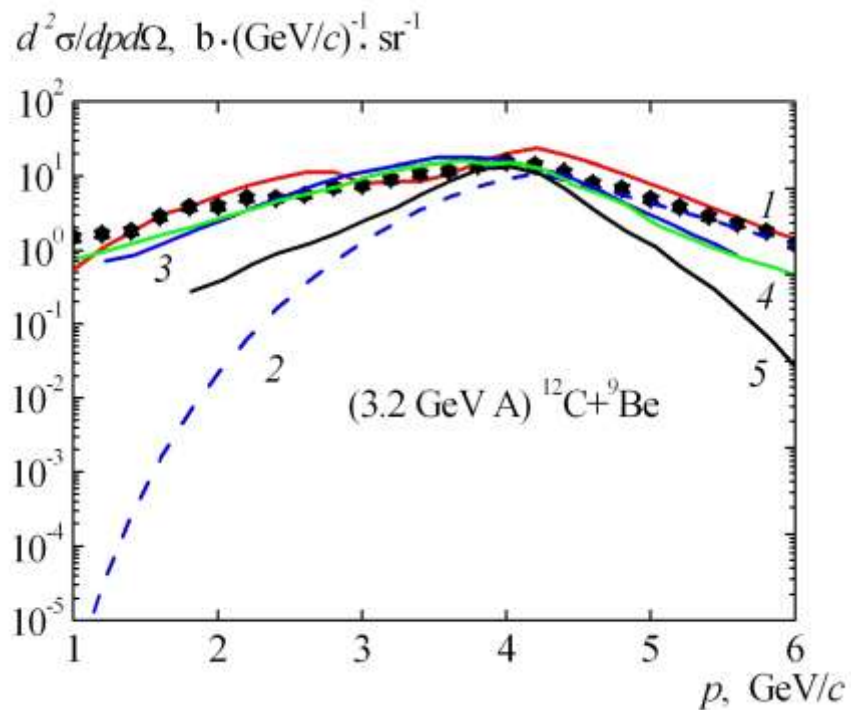


$^{12}\text{C}+^9\text{Be}$ 950 MeV/nucleon.
(protons) $3,5^0$

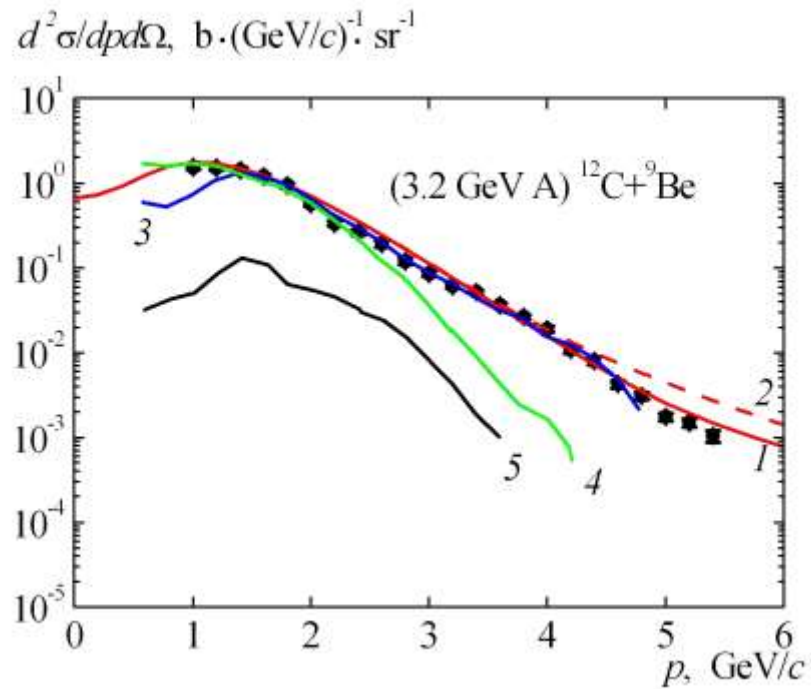


$^{12}\text{C}+^9\text{Be}$ 2000 MeV/nucleon.
(protons) $3,5^0$

**$^{12}\text{C}+^9\text{Be}$ 3.2 GeV/nucl.
(protons) 3,5⁰**

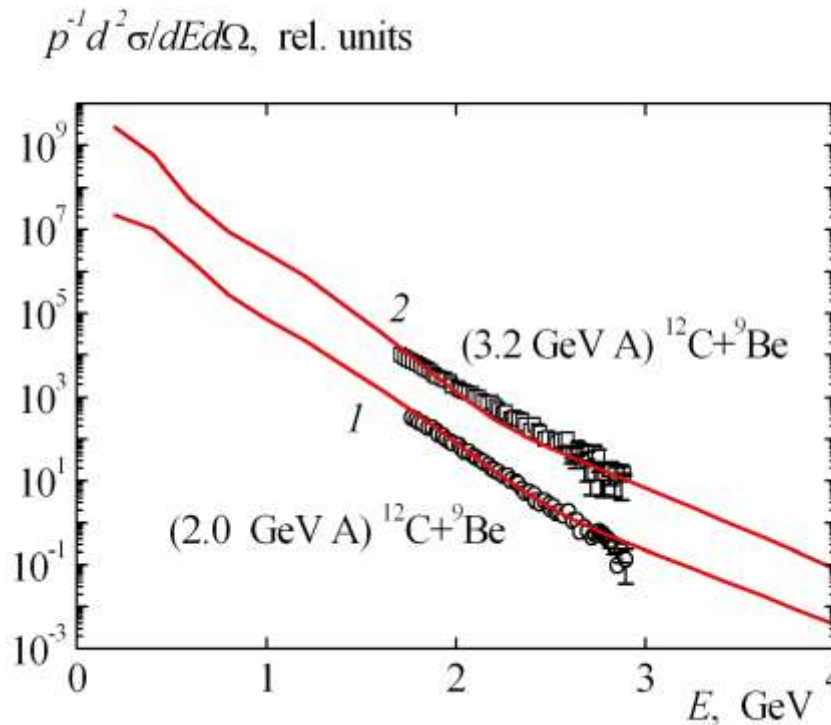


**$^{12}\text{C}+^9\text{Be}$ 3.2 GeV/nucl.
(negative pions) 3,5⁰**

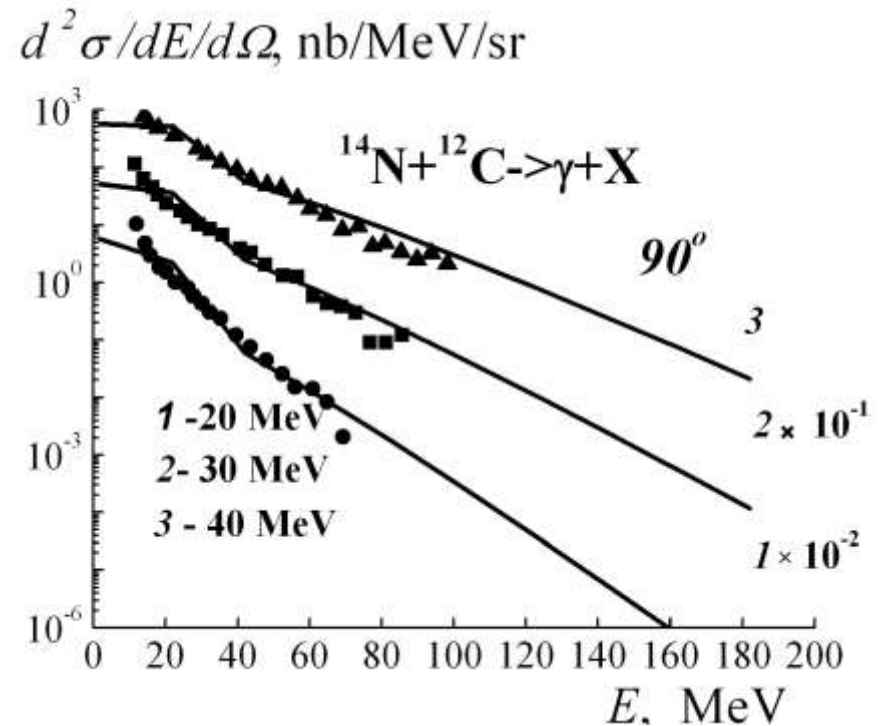


Hard photons

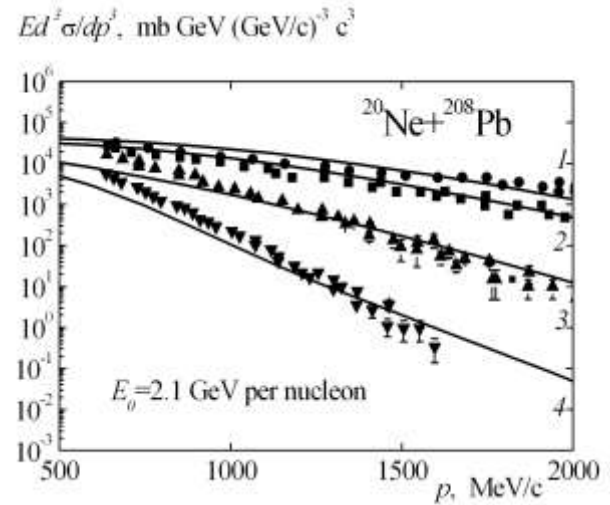
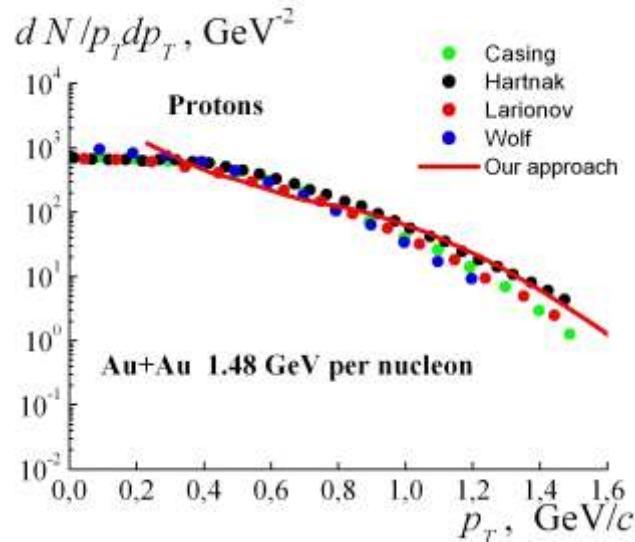
$^{12}\text{C}+^9\text{Be}$ (photons) 38°



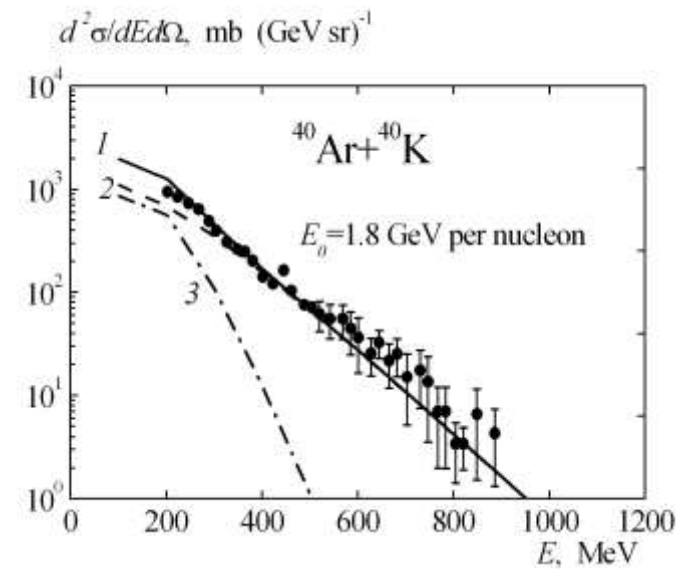
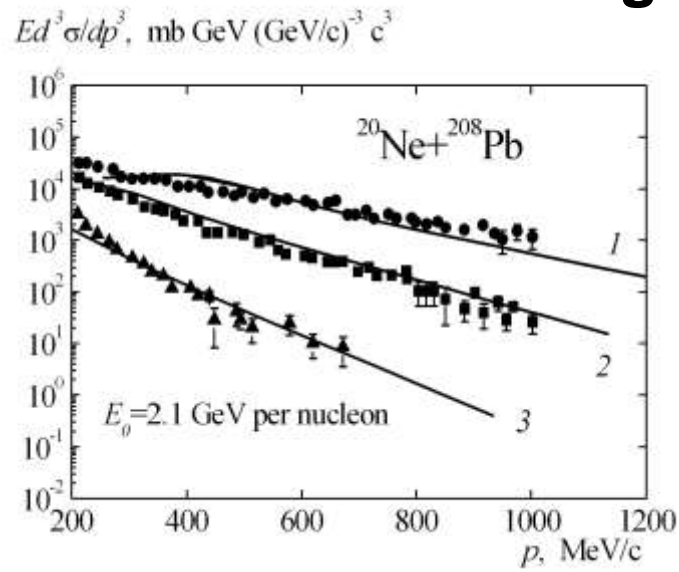
$^{14}\text{N}+^{12}\text{C}$ (photons) 90°



Protons



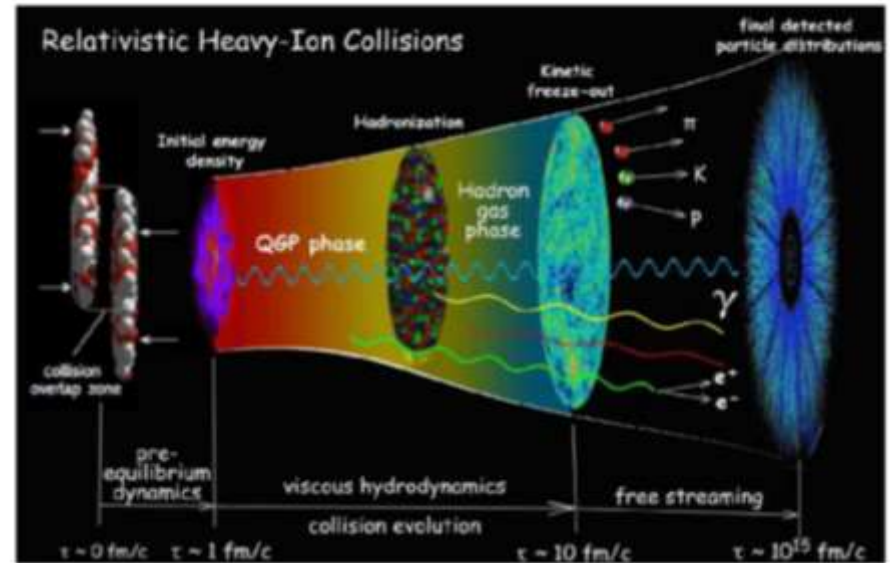
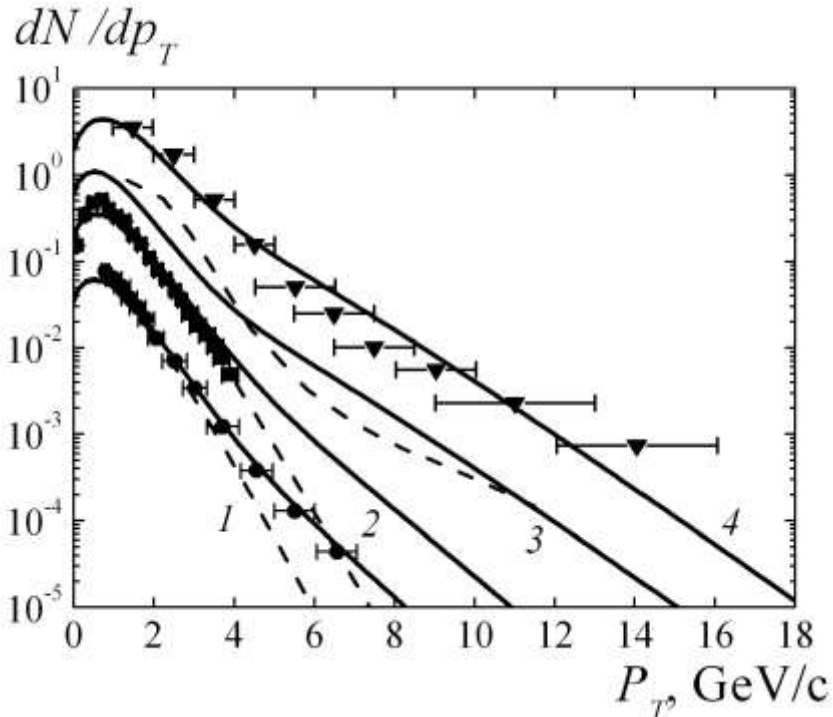
Negative pions



LHC

$$\frac{dN}{dp_T} = CTp_T \exp\left(-\frac{\sqrt{m^2 + p_T^2} - m}{T}\right)$$

$$T = \left(\frac{E_0}{g_Q V_R} 10^9\right)^{1/4} \quad g_Q = \left(2 \times 8 + \frac{7}{8} 2 \times 2 \times 3 \times 3\right) = 47$$

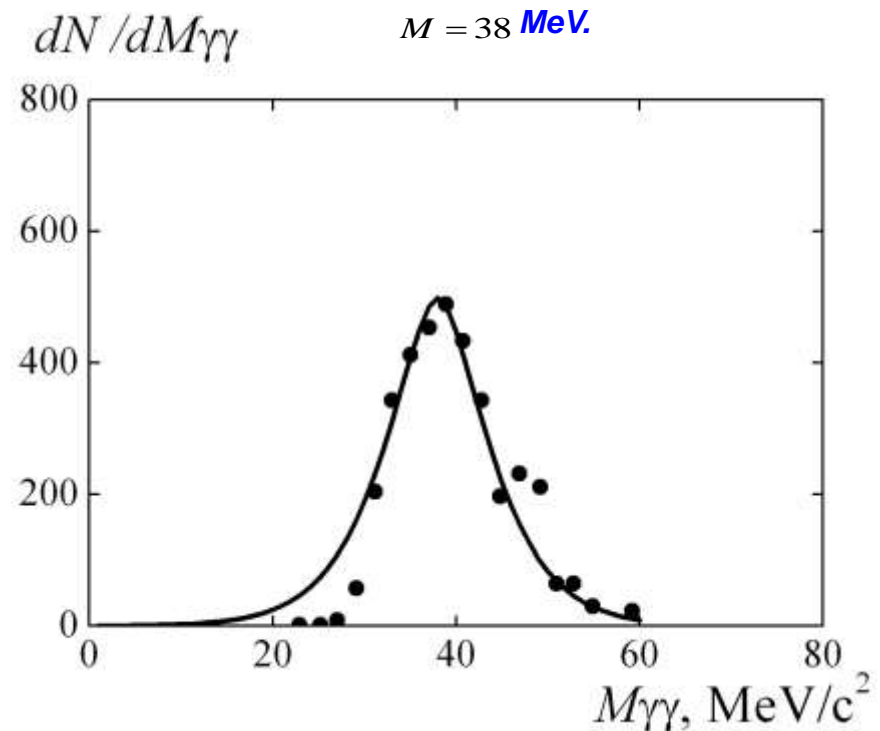
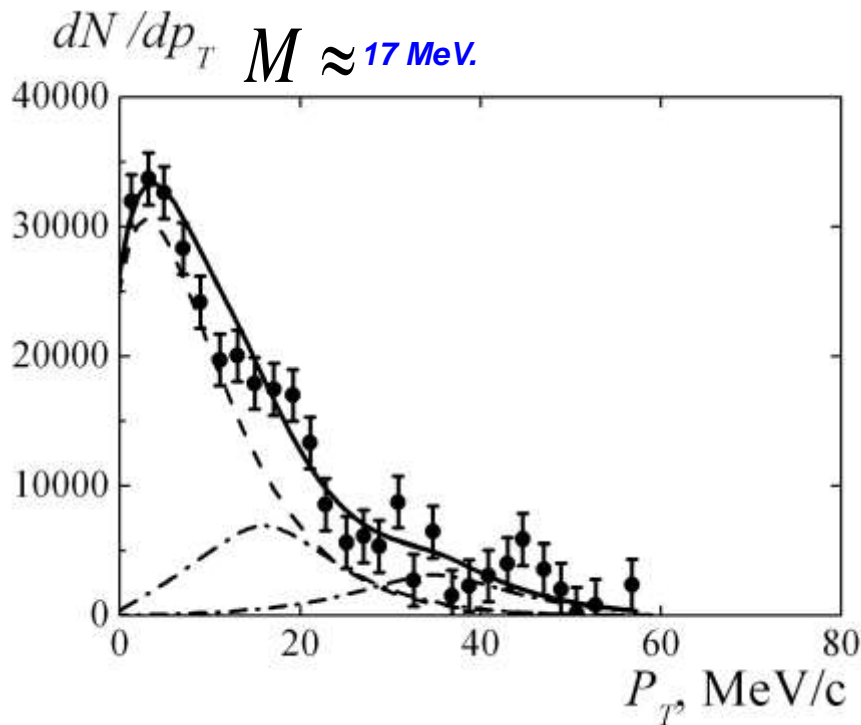


Dark matter for soft photons

$$p + p \rightarrow 2\gamma + X$$

$$M^2 = 2\pi\rho n$$

$$p + C \rightarrow 2\gamma + X$$



A. Belogianni et al., Phys. Lett. B548, 129 (2002)

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Conclusions

- Thus, in this paper, the idea of using of a non-equilibrium equation of state in the hydrodynamic approach with hot spot to describe the high-momentum proton, pion and photon spectra emitted in heavy-ion collisions over a wide energy range has been further developed.
- The experimental shoulder in the cross section for the production of protons in the cumulative region is reproduced by our calculations, and sometimes by cascade models. The cumulative pions and gamma-quanta are reproduced by ours too.
- The existence of the X17 boson mass, equal to 17 MeV, and X38, equal to 38 MeV, is substantiated based on the electromagnetic tube when two-dimensional QCD_2 and QED_2 are combined. The search for new particles possible candidates for the role of light particles of dark matter is very relevant

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THANK YOU !