Multiplicity distributions and combinants in multi-pomeron exchange model

Evgeny Andronov, Vladimir Kovalenko,
Andrey Putchkov, Vladimir Vechernin

Saint-Petersburg State University

The LXXII International Conference "Nucleus-2022: Fundamental problems and applications"

Moscow, 11-16 July 2022

Multiplicity distribution and combinants

- Generating function for multiplicity distribution:

$$G(t) = \sum_{N=0}^{\infty} P(N)\, t^N$$

- Generating function for combinants:

$$F(t) = \sum_{j=0}^{\infty} C^*(j)\, t^j \,, \qquad \text{where} \qquad F(t) = \ln G(t)$$

- Recurrence relation for combinants:

$$N\, P(N) = \sum_{j=1}^{N} j\, C^*(j)\, P(N-j)$$

Modified combinants

$$C(j) \equiv \frac{j+1}{\langle N\rangle} C^*(j+1)\,, \quad \text{where} \quad \langle N\rangle = \sum_{N=1}^{\infty} N\, P(N)$$

- Recurrence relation for modified combinants:

$$(N+1)\,P(N+1) = \langle N\rangle \sum_{j=0}^{N} C(j)\,P(N-j)$$

Relationships of this type or even simpler ones:

$(N+1)\,P(N+1) = g(N)\,P(N)$, where *g(N)* is a linear function,

are found in many models (clans, cascade models).

See e.g. formula (32) in

M.A. Braun, C. Pajares, V.V. Vechernin, Physics Letters B 493 (2000) 54–64,
“On the forward–backward correlations in a two-stage scenario”.

$$X(j) \equiv \langle N\rangle C(j) = (j+1)\,C^*(j+1)$$

Using the recurrence relation to calculate combinants

$$X(N) = (N+1)\frac{P(N+1)}{P(0)} - \sum_{j=0}^{N-1} X(j)\frac{P(N-j)}{P(0)}$$

Explicit solution for the first few $X(j) \equiv \langle N \rangle C(j) =$

$$= (j+1)\, C^*(j+1)$$

$$\overline{P}(N) \equiv \frac{P(N)}{P(0)}$$

$$X(0) = \overline{P}(1) = \frac{P(1)}{P(0)}$$

$$X(1) = 2\overline{P}(2) - \overline{P}^2(1) = 2\frac{P(2)}{P(0)} - \left(\frac{P(1)}{P(0)}\right)^2$$

$$X(2) = 3\overline{P}(3) - 3\overline{P}(1)\overline{P}(2) + \overline{P}^3(1)$$

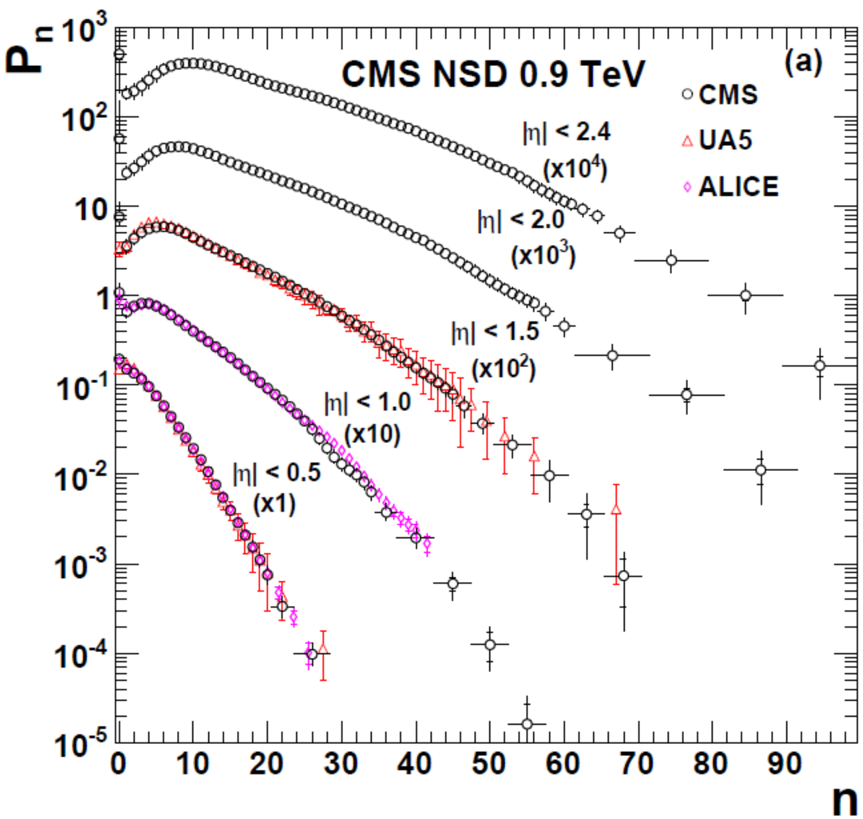
$$X(3) = 4\overline{P}(4) - 4\overline{P}(1)\overline{P}(3) - 2\overline{P}^2(2) + 4\overline{P}^2(1)\overline{P}(2) - \overline{P}^4(1)$$

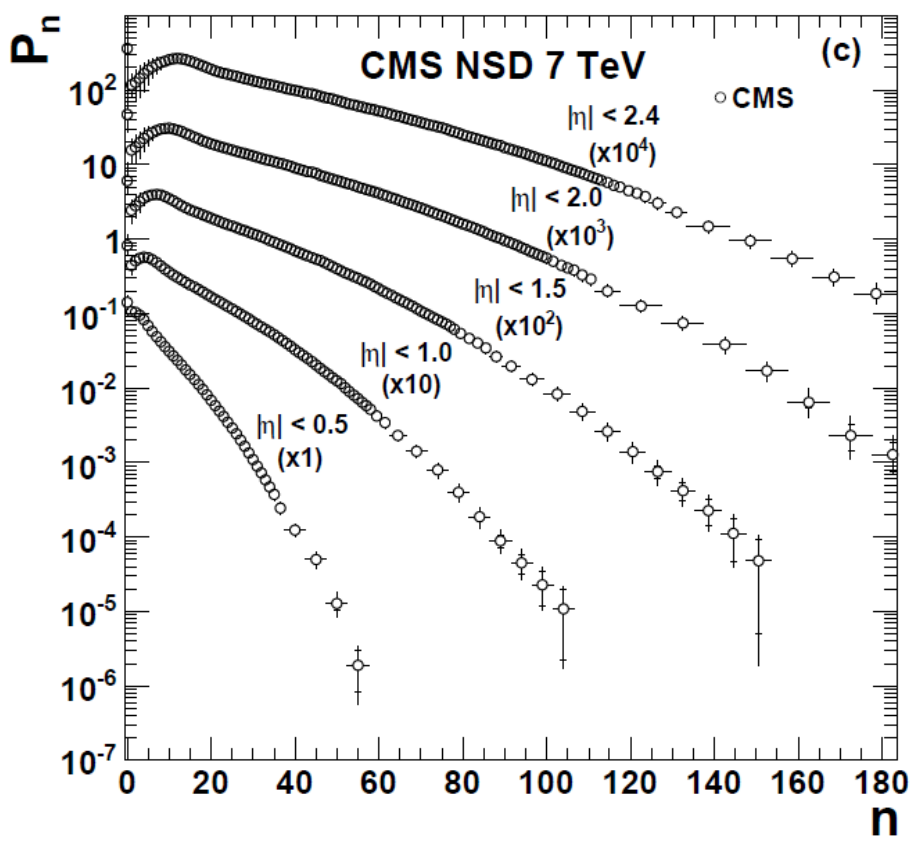
Why combinants?

Multiplicity distributions in pp collisions at LHC energies

ALICE Collaboration, Eur.Phys.J.C68, 89 (2010); ibid C68, 345(2010),

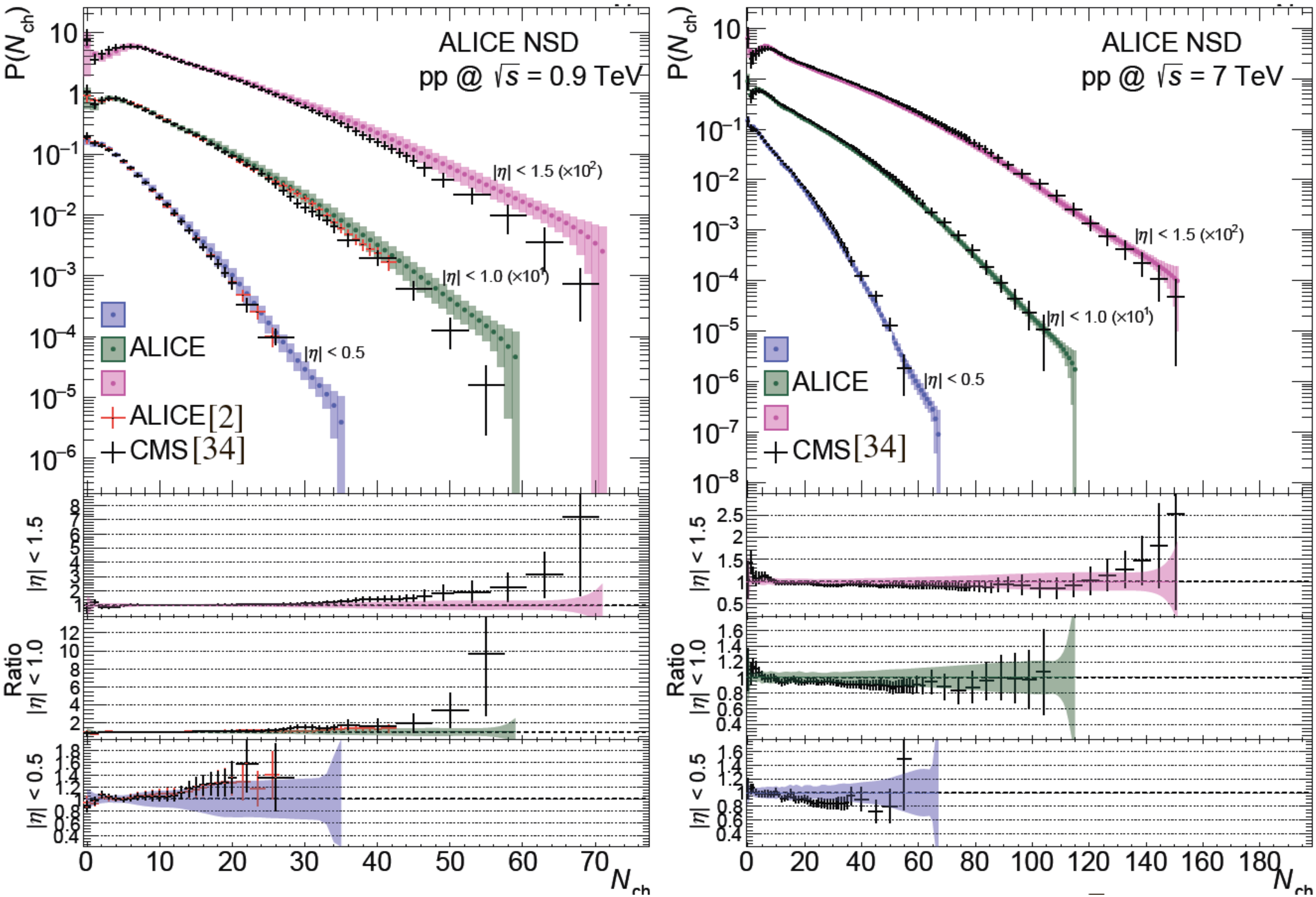
CMS Collaboration, JHEP 01, 079 (2011).





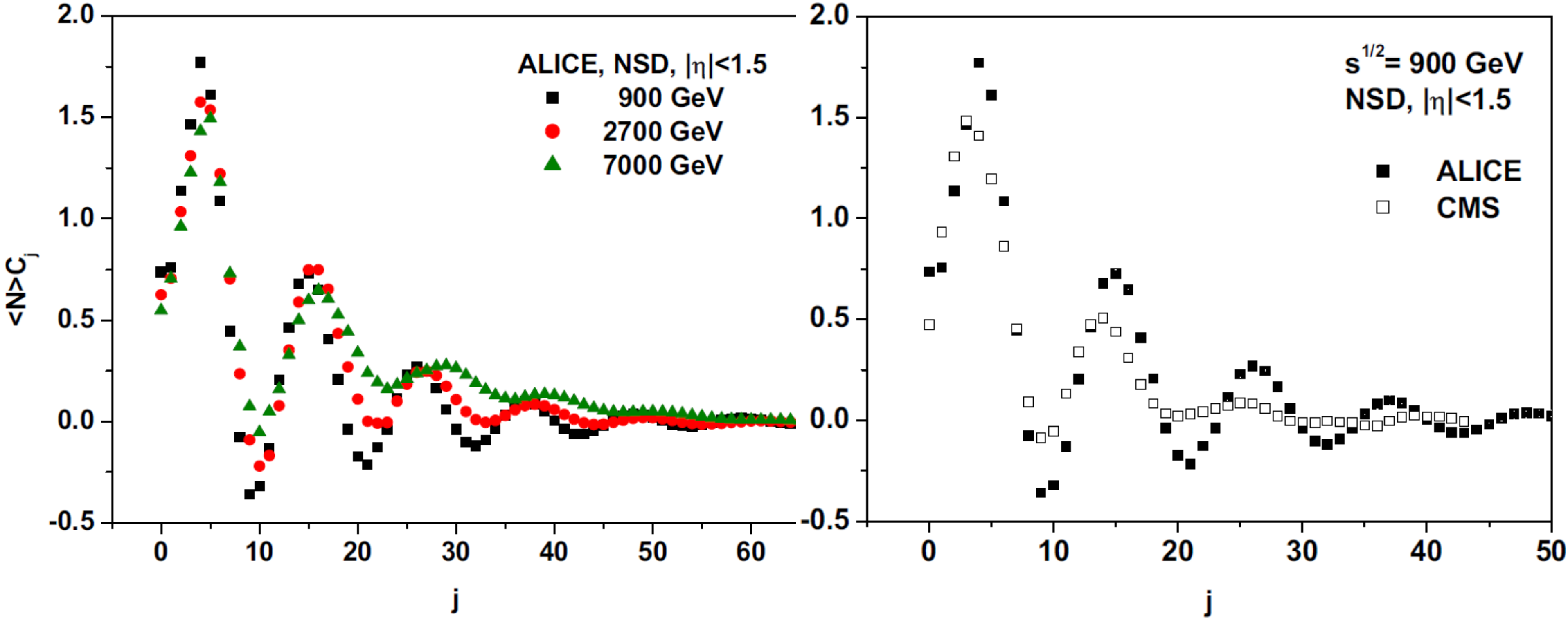
Multiplicity distributions in pp collisions at LHC energies

ALICE Collaboration, Eur.Phys.J.C77, 33 (2017).



Combinants extracted from the experimental data

G. Wilk, Z. Włodarczyk, J. Phys. G 44, 015002 (2017).



ALICE Collaboration, Eur.Phys.J.C77, 33 (2017),

CMS Collaboration, JHEP 01, 079 (2011).

Multi-Pomeron Exchange Model

N. Armesto, D.A. Derkach, G.A. Feofilov, Phys. Atom. Nucl. 71, 2087 (2008),
E.O. Bodnya, V.N. Kovalenko, A.M. Puchkov, G.A. Feofilov, AIP Conf.Proc. 1606, 273 (2015),
V.N. Kovalenko, A.M. Puchkov, G.A. Feofilov, Bull. Russ. Acad. Sci. Phys. 80, 966 (2016),
G. Feofilov, V. Kovalenko, A. Puchkov, Eur. Phys. J.: Web of Conf. 171, 18003 (2018),
E.V. Andronov, V.N. Kovalenko, Theor.Math.Phys. 200, 1282 (2019),
V. Kovalenko, G. Feofilov, A. Puchkov, F. Valiev, Universe 8, 246 (2022).

$$P(N) = C(z) \sum_{N_{pom}} \frac{1}{z \cdot N_{pom}} \left(1 - e^{-z} \sum_{l=0}^{N_{pom}-1} \frac{z^l}{l!} \right) \cdot P_{N_{pom}}(N)$$

$$z = \frac{2C\gamma s^{\Delta}}{R^2 + \alpha' \log(s)}, \ \Delta = 0.139, \ \alpha' = 0.21 \text{ GeV}^{-2}, \ \gamma = 1.77 \text{ GeV}^{-2}, \ R_0^2 = 3.18 \text{ GeV}^{-2}, \ C = 1.5$$

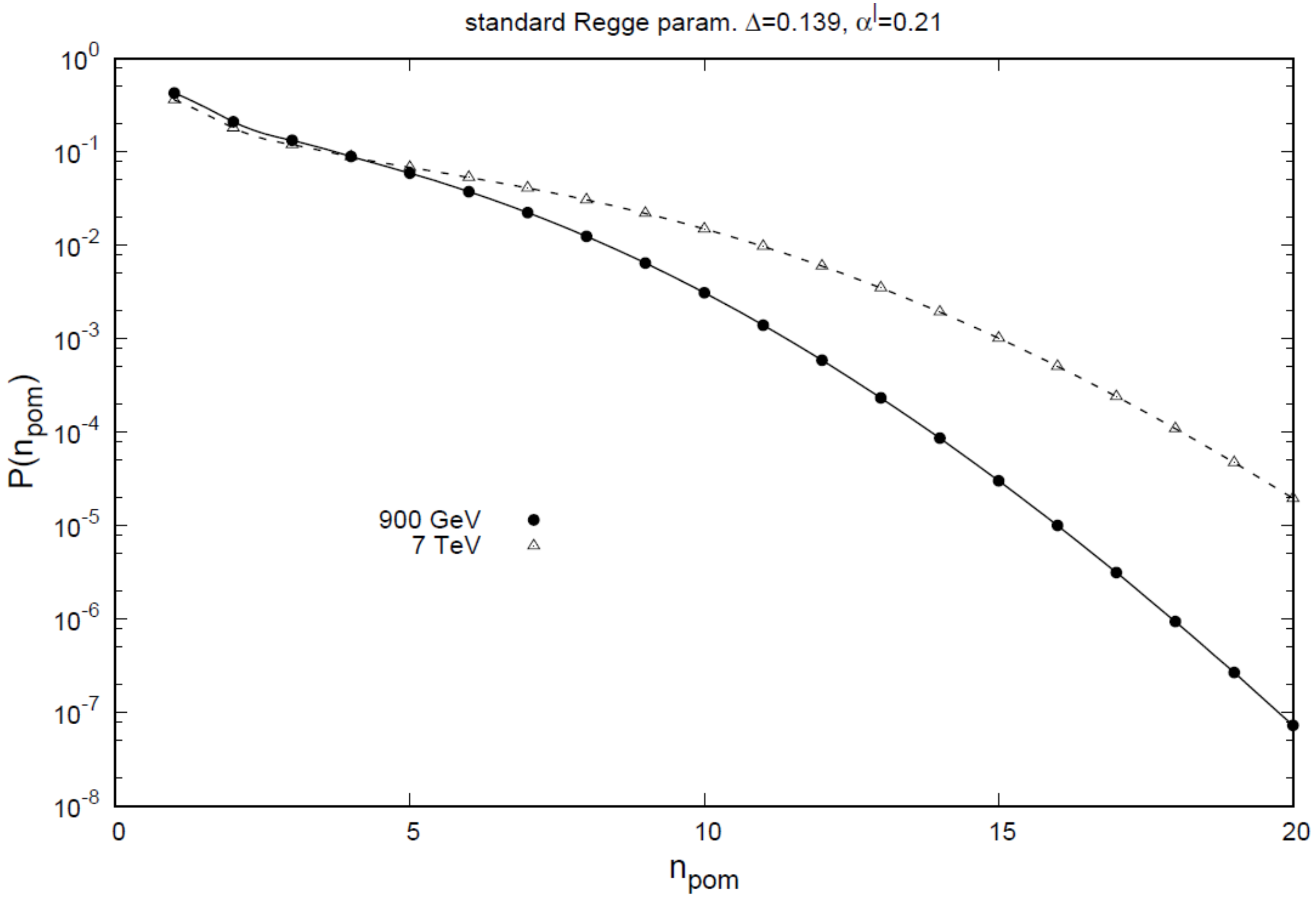
$P_{N_{pom}}(N)$ was taken to be Poisson distribution with mean $2 \cdot N_{pom} \cdot \delta\eta \cdot k(\sqrt{s})$ where $k = 0.255 + 0.0653 \cdot ln\sqrt{s}$

Regge parameters are taken from:
G. H. Arakelyan, A. Capella, A. B. Kaidalov, Yu. M. Shabelski, EPJC 26, 81 (2002).

Each cut pomeron corresponds to a pair of quark-gluon strings.

An increase in the multiplicity density with energy is explained by an increase in the mean number of cut pomerons and a growth of the average multiplicity from a single string.

Distribution in number of cut pomerons



Regge parameters are taken from:

G. H. Arakelyan, A. Capella, A. B. Kaidalov, Yu. M. Shabelski, EPJC 26, 81 (2002).

Modeling a particle distribution from *n* cut pomerons

$$P(N) = \sum_{N_{pom}=1}^{\infty} P(N_{pom})\, P_{N_{pom}}(N) = \sum_{n=1}^{\infty} P(n)\, P_n(N)\,, \qquad n \equiv N_{pom}$$

$$\langle N \rangle_n = 2n\, \delta\eta\, k(s) = 2n\, \langle N \rangle_{str} \equiv 2n\, \mu_{str}$$

Poisson Distribution

$$P_n(N) = e^{-\langle N \rangle_n} \frac{\langle N \rangle_n{}^N}{N!}\,, \qquad P_{str}(N) = e^{-\mu_{str}} \frac{\mu_{str}^N}{N!}$$

Negative Binomial Distribution

$$P_n(N) = \frac{p^N}{q^{N+\kappa}} \frac{\Gamma(N+\kappa)}{\Gamma(\kappa)\, N!}\,, \qquad g_{NBD}(t) = (q - p\,t)^{-\kappa}$$

$$q = \omega \equiv \frac{\langle N^2 \rangle_n - \langle N \rangle_n^2}{\langle N \rangle_n} = \frac{\langle N^2 \rangle_{str} - \langle N \rangle_{str}^2}{\langle N \rangle_{str}} \equiv \omega_{str}$$

=> true also for merged, but identical strings

$$p = q - 1 = \omega_{str} - 1\,, \qquad \kappa = \frac{\langle N \rangle_n}{p} = 2n \frac{\mu_{str}}{\omega_{str} - 1}$$

Fluctuation in the number of particles from one string

V. Vechernin, Nucl.Phys.A 939, 21 (2015)

In the presence of *correlations* between particles, there can be *no Poisson distribution* from the source.

$$\frac{\langle N^2\rangle_{str} - \langle N\rangle_{str}^2}{\langle N\rangle_{str}} \equiv \omega_{str} = 1 + \mu_{str} J \,, \qquad J \equiv \frac{1}{\delta\eta^2} \int_{\delta\eta} d\eta_1 \int_{\delta\eta} d\eta_2 \, \Lambda(\eta_1 - \eta_2)$$

The pair correlation function of a single string:

$$\Lambda(\eta_1, \eta_2) \equiv \frac{\lambda_2(\eta_1, \eta_2)}{\lambda(\eta_1)\lambda(\eta_2)} - 1 = \frac{\lambda_2(\eta_1 - \eta_2)}{\mu_0^2} - 1 = \Lambda(\eta_1 - \eta_2)$$

The one- and two-particle rapidity distributions of particles from one string decay:

$$\lambda(\eta) \equiv \frac{dN}{d\eta} = \frac{\mu_{str}}{\delta\eta} = \mu_0 = k(s) \,, \qquad \lambda_2(\eta_1, \eta_2) \equiv \frac{d^2N}{d\eta_1\, d\eta_2} = \lambda_2(\eta_1 - \eta_2)$$

The parametrization for the pair correlation function of a single string $\Lambda(\Delta\eta, \Delta\phi)$ **using the data from** *ALICE Collaboration, JHEP 05, 097 (2015), Forward-backward multiplicity correlations in pp collisions at 0.9, 2.76 and 7 TeV*

Then integrating over azimuth we find :

$$\Lambda(\Delta\eta) = \frac{1}{\pi} \int_0^{\pi} \Lambda(\Delta\eta, \Delta\phi)\, d\Delta\phi$$ **well approximated by** $$\Lambda(\Delta\eta) = \Lambda_0 \, e^{-\frac{|\Delta\eta|}{\eta_{corr}}}$$

V. Vechernin, EPJ Web Conf. 191, 04011 (2018),
E. Andronov, V. Vechernin, Eur. Phys. J. A 55, 14 (2019).

Fluctuation in the number of particles from one string

For the exponential pair correlation function, the integral can be calculated explicitly:

S. Belokurova, Phys. Part. Nucl. 53, 154 (2022)

$$\omega_{str} = 1 + \mu_{str} J \ , \qquad J = \frac{2\Lambda_0\, \eta_{corr}}{(\delta\eta)^2}\left[\delta\eta - \eta_{corr}\left(1 - e^{-\delta\eta/\eta_{corr}}\right)\right]$$

Then we find:

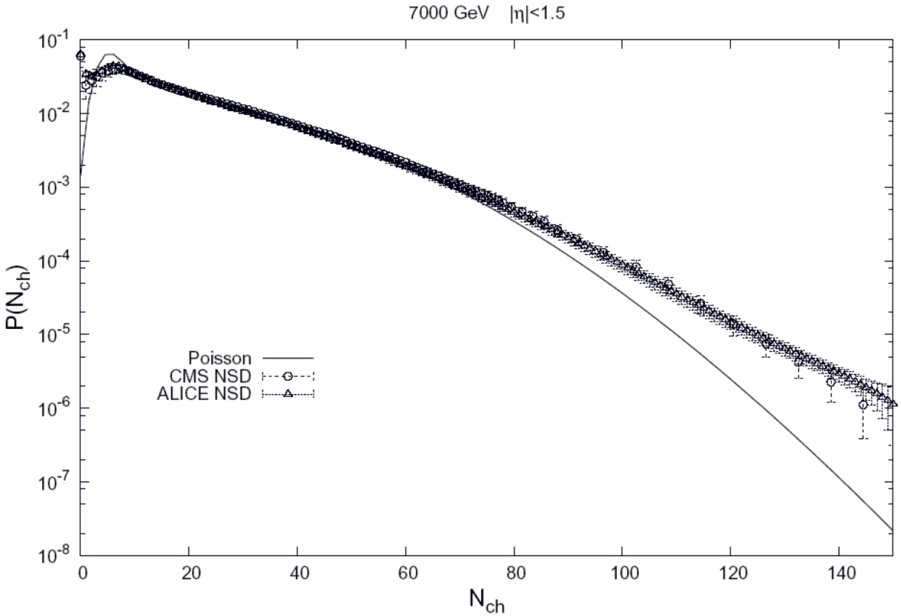
$$\frac{\langle N^2\rangle_{str} - \langle N\rangle^2_{str}}{\langle N\rangle_{str}} \equiv \omega_{str}$$

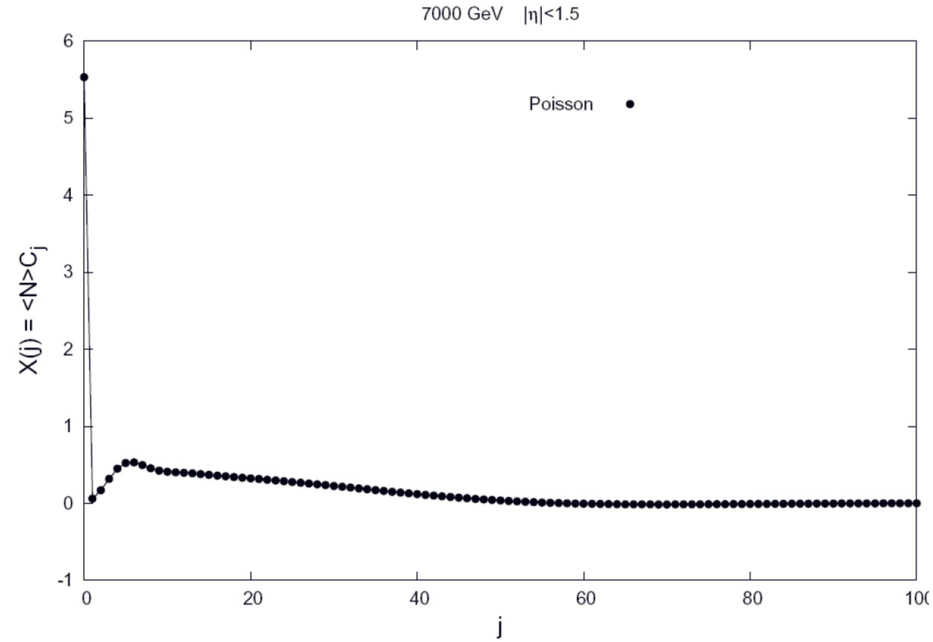
$\sqrt{s}$, TeV	0.9	7.0
$\delta\eta = 3$	3.1	3.5
$\delta\eta = 4.8$	3.6	4.0

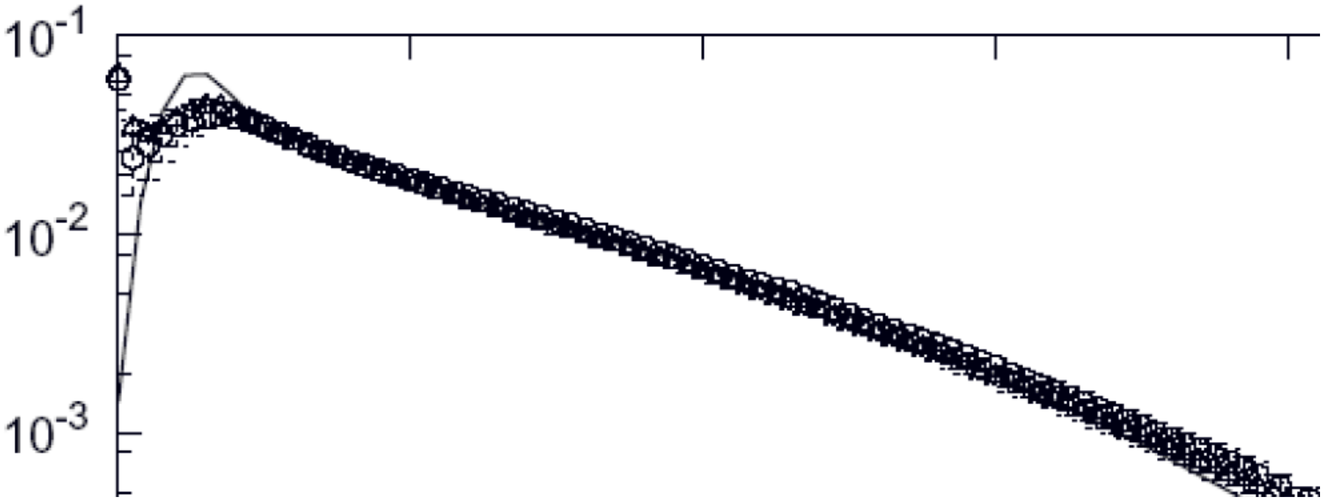
Recall that in the multipomeron exchange model:

$$\mu_{str} = k(s)\,\delta\eta$$

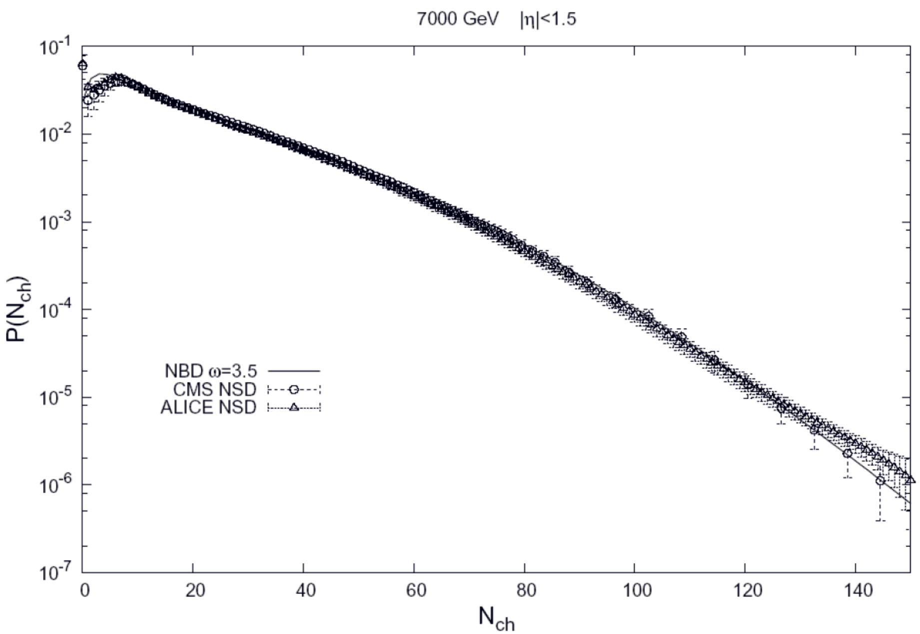
Assuming a Poisson distribution of particles from one string

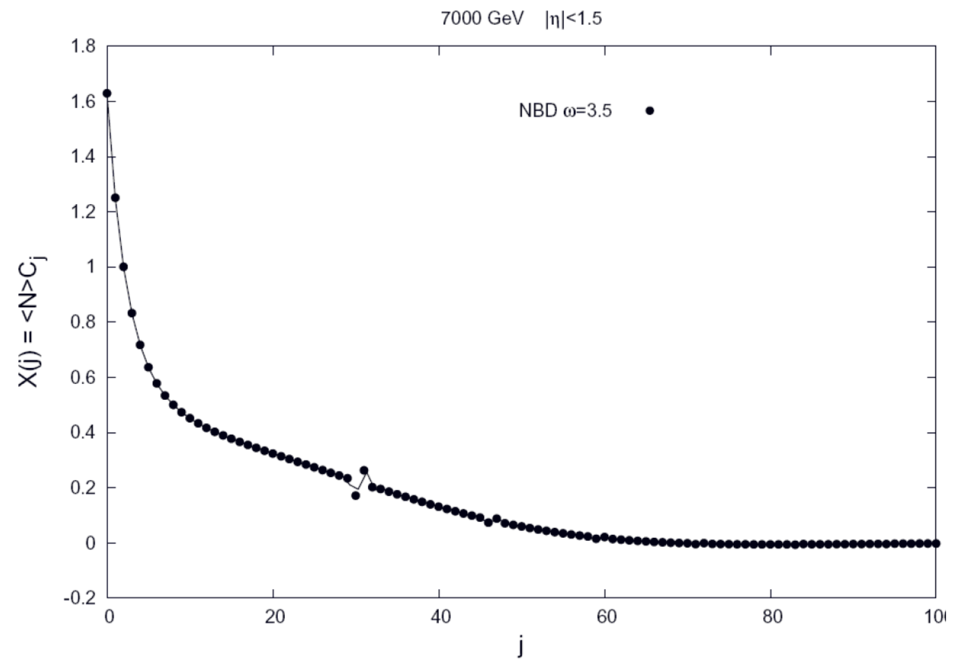


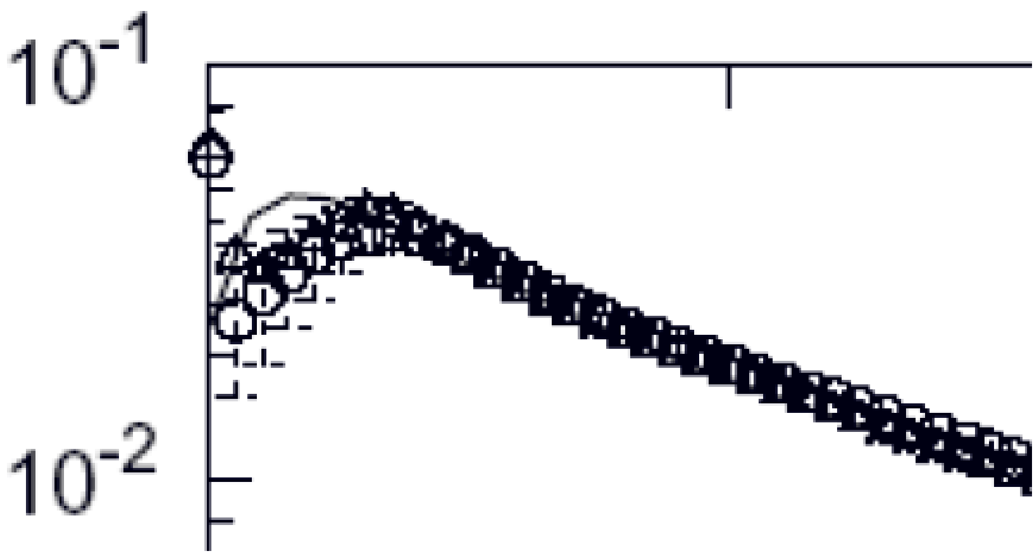




Assuming a Negative Binomial distribution from one string







Assuming a Gaussian distribution for $P_n(N)$

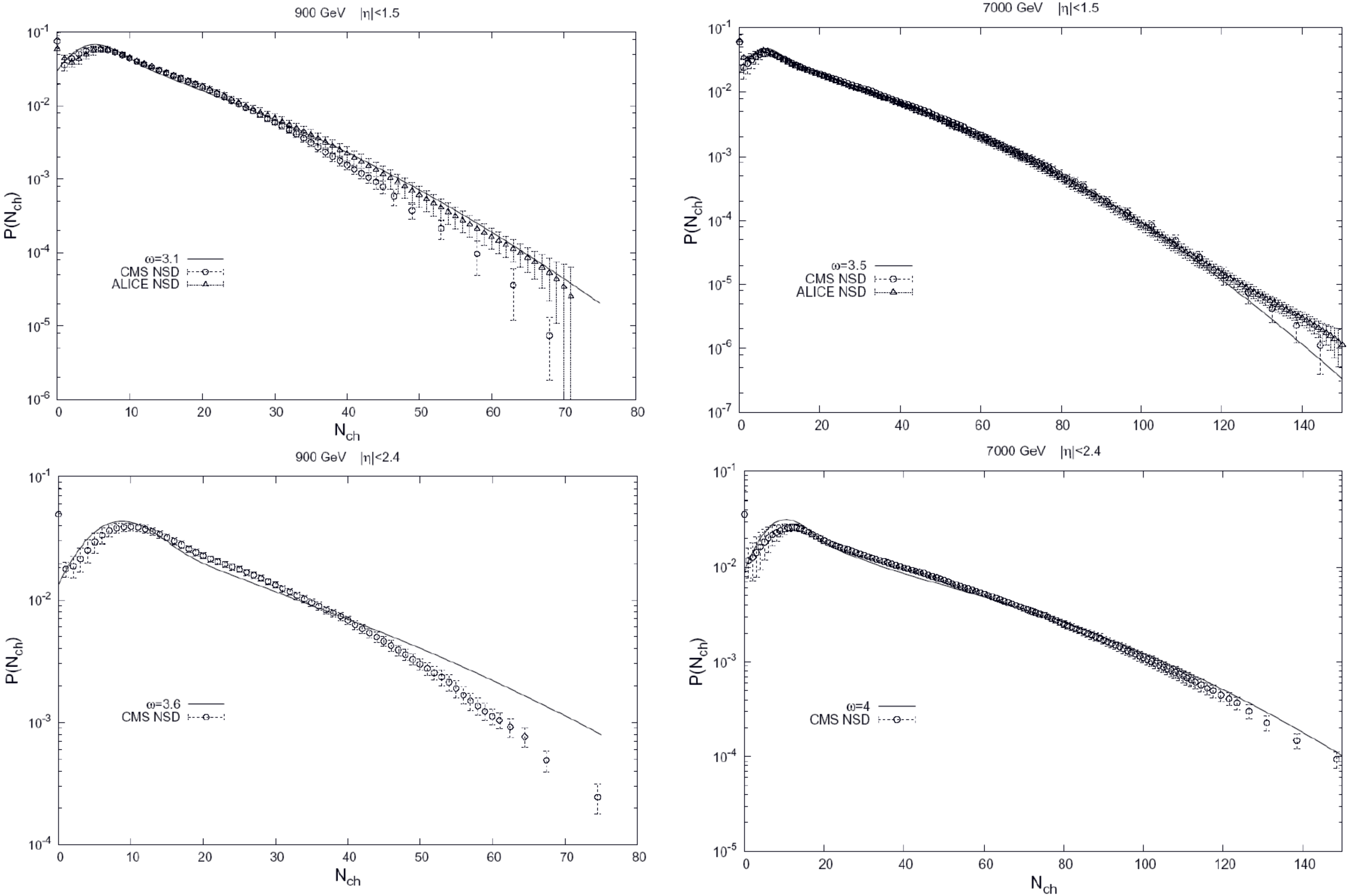
$$P_n(N) = C \exp\left[-\frac{(N - 2n\mu_{str})^2}{2\omega_{str}\, 2n\mu_{str}}\right], \qquad \sum_{N=0}^{\infty} P_n(N) = 1$$

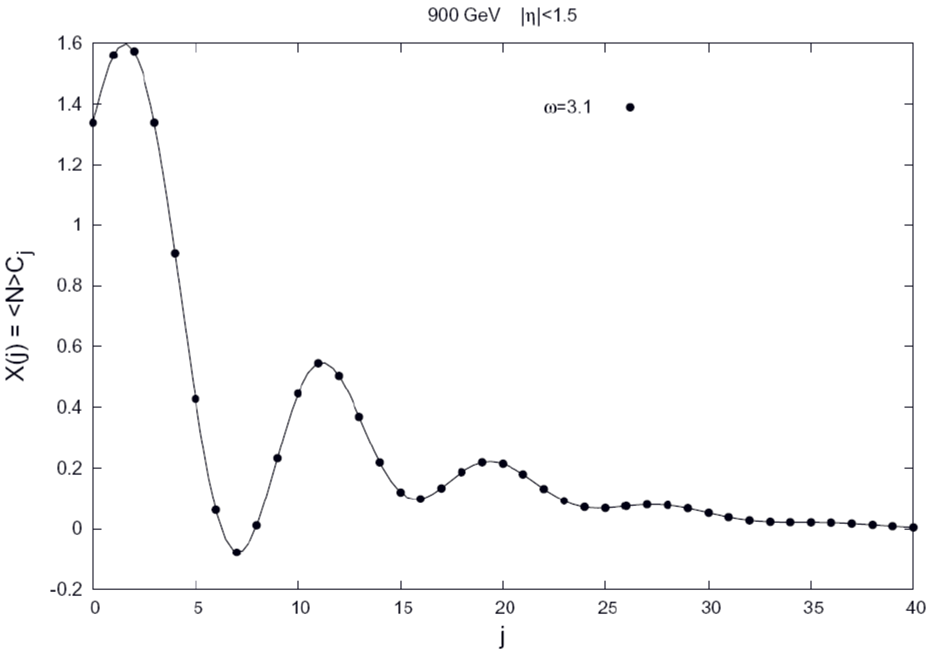
$$C^{-1} = \sum_{N=0}^{\infty} \exp\left[-\frac{(N - 2n\mu_{str})^2}{2\omega_{str}\, 2n\mu_{str}}\right]$$

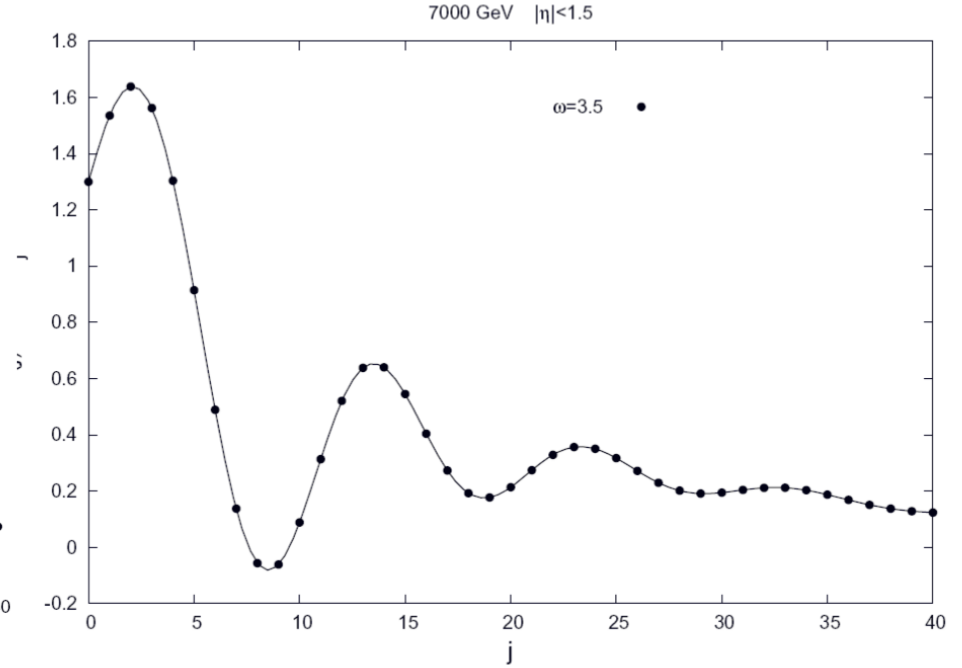
For $2n\mu_{str} \gg 1$ we have

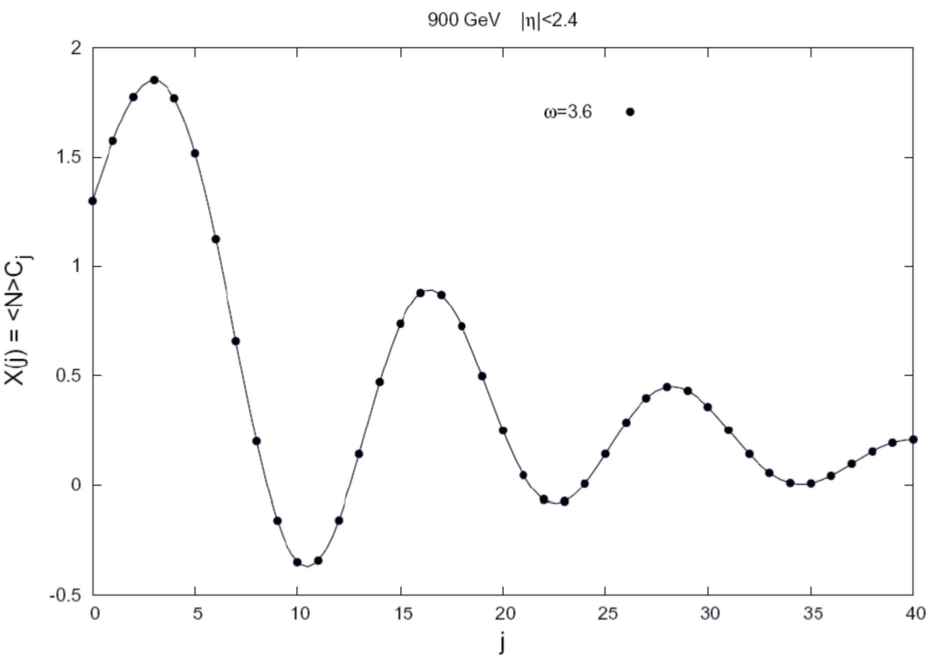
$$\langle N \rangle_n \equiv \sum_{N=1}^{\infty} N\, P_n(N) \to 2n\mu_{str}$$

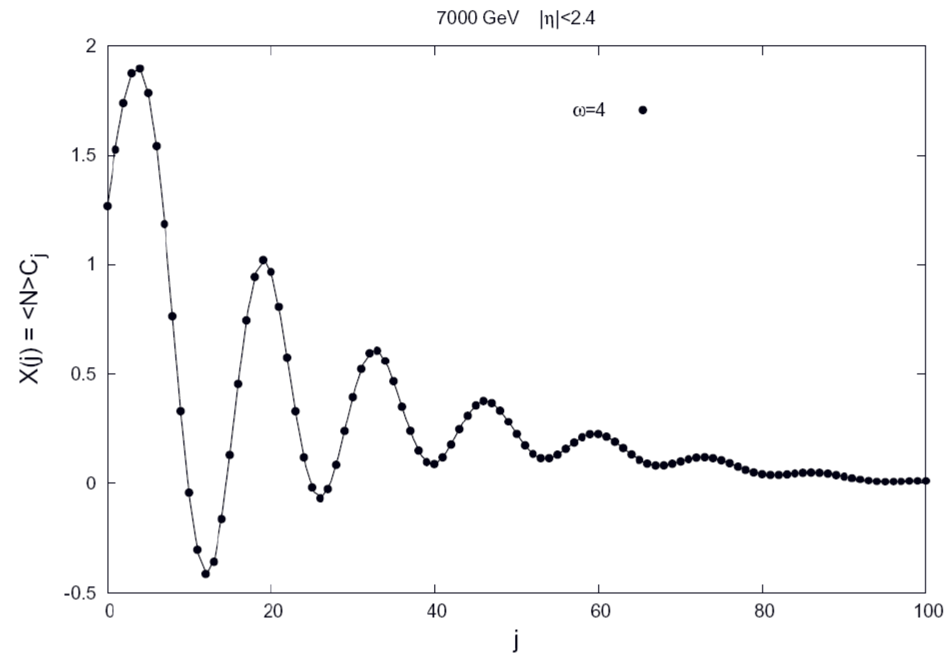
$$\omega_n[N] \equiv \frac{\langle N^2 \rangle_n - \langle N \rangle_n^2}{\langle N \rangle_n} \to \omega_{str}$$


900 GeV |η|<1.5
ω=3.1
CMS NSD
ALICE NSD
P(N_ch)
N_ch
7000 GeV |η|<1.5
ω=3.5
CMS NSD
ALICE NSD
P(N_ch)
900 GeV |η|<2.4
ω=3.6
CMS NSD
P(N_ch)
N_ch
7000 GeV |η|<2.4
ω=4
CMS NSD
P(N_ch)
N_ch

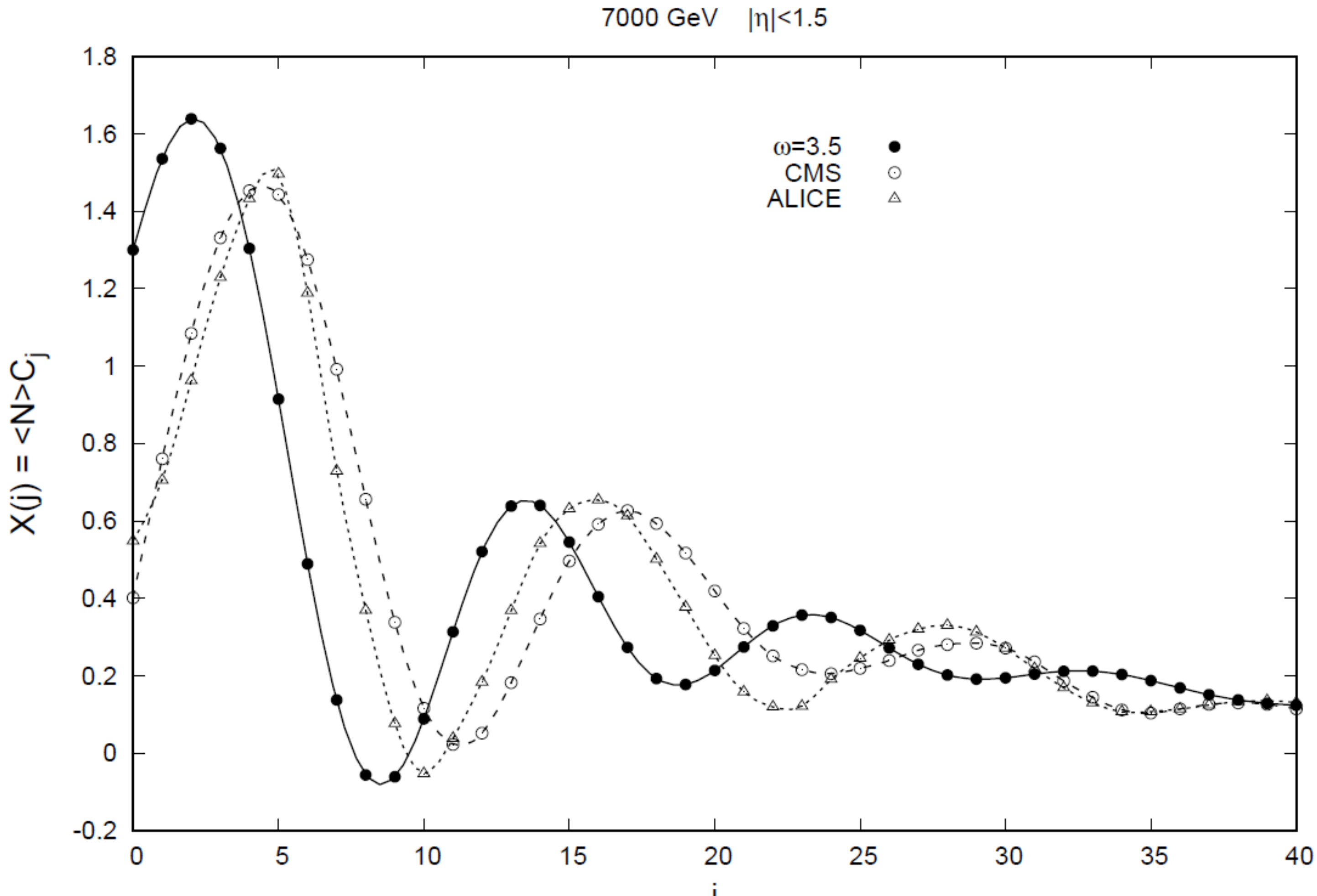

900 GeV |η|<1.5
ω=3.1
X(j) = <N>C_j
j


7000 GeV |η|<1.5
ω=3.5
j

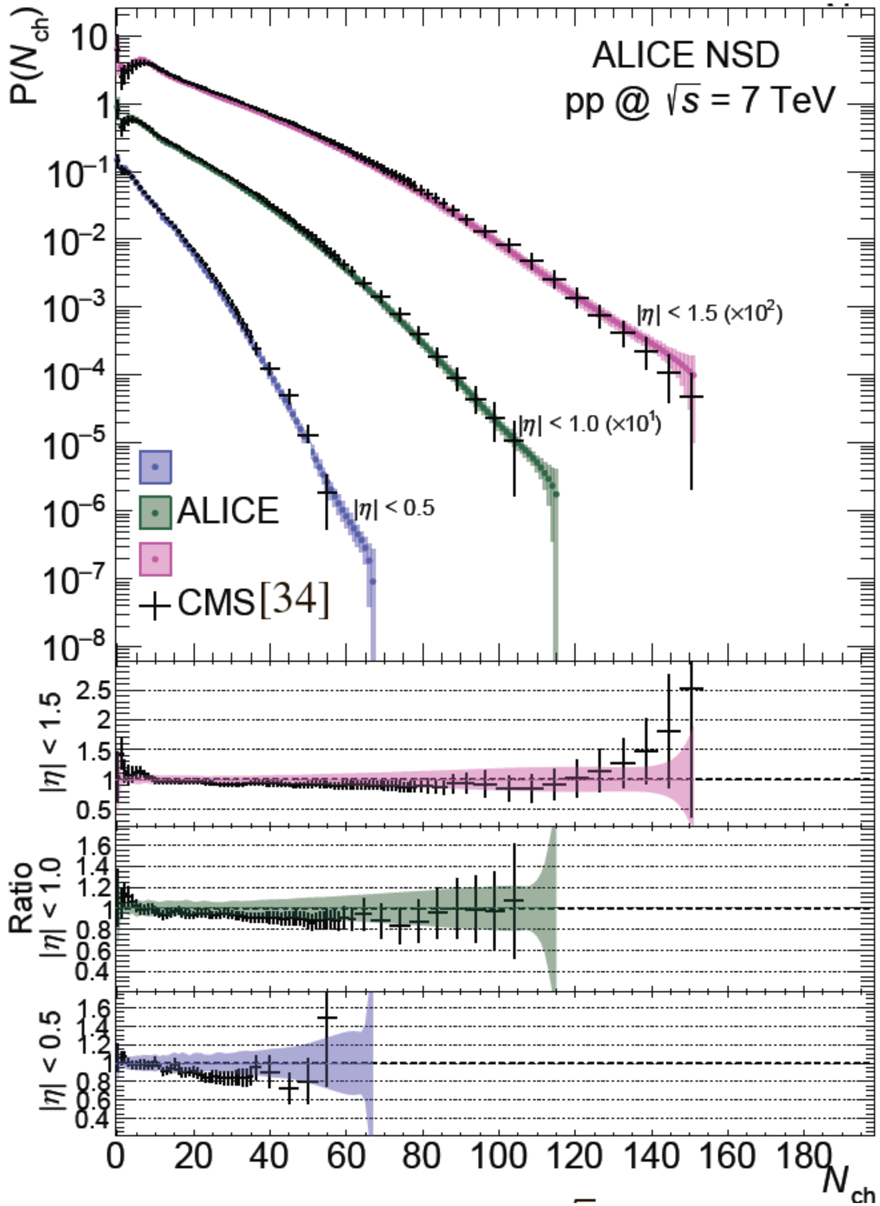

900 GeV |η|<2.4
ω=3.6
X(j) = <N>C_j
j

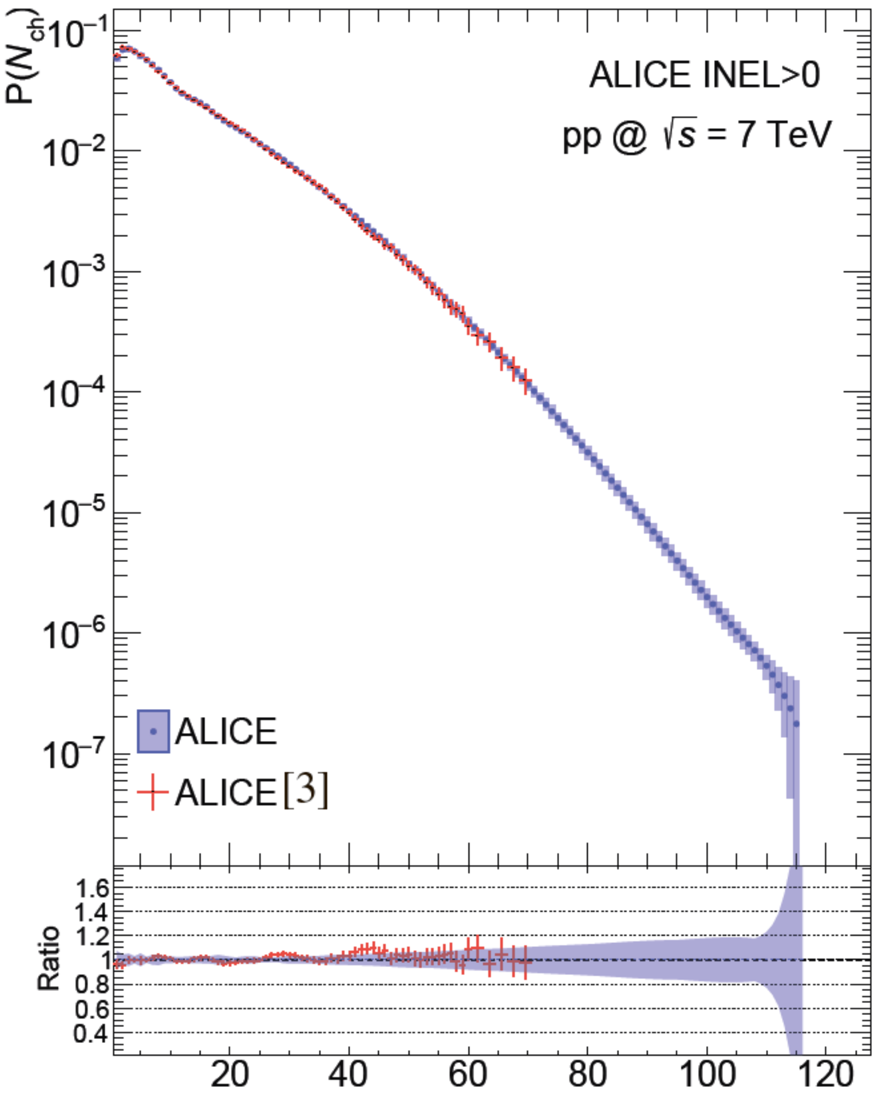

7000 GeV |η|<2.4
ω=4
X(j) = <N>C_j
j

Comparison to the pp experimental data



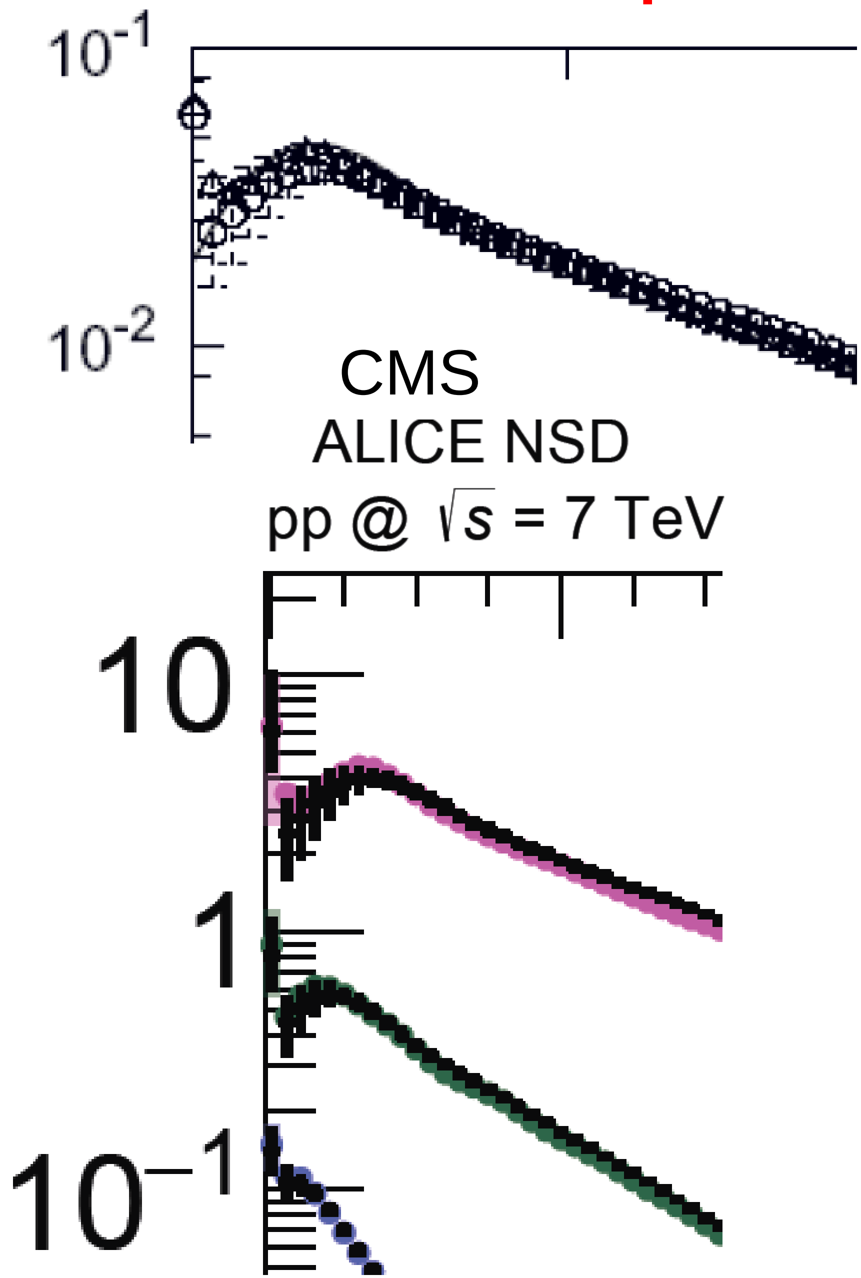
ALICE Collaboration, Eur.Phys.J.C68, 89 (2010); ibid C68, 345(2010), CMS Collaboration, JHEP 01, 079 (2011), ALICE Collaboration, Eur.Phys.J.C77, 33 (2017)



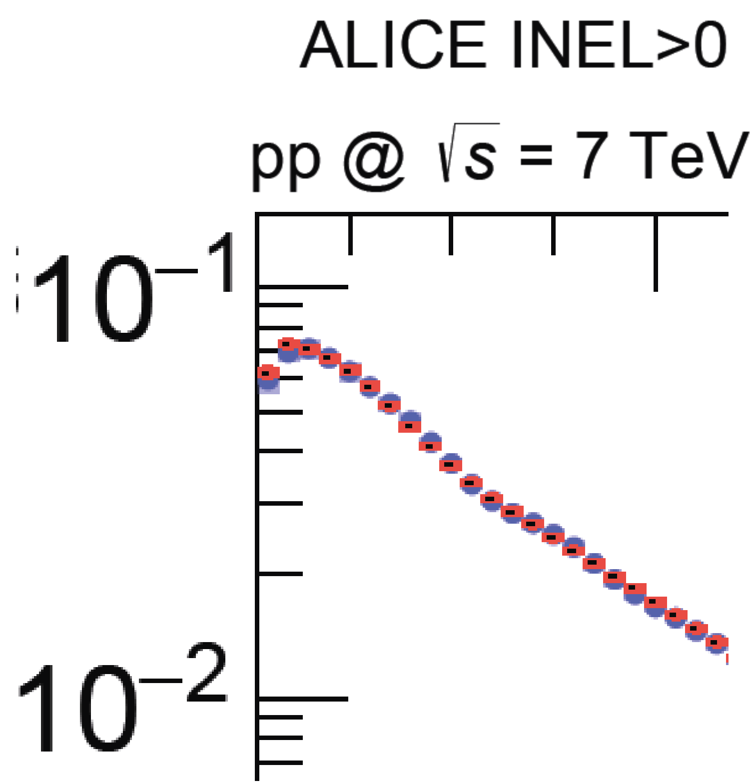


ALICE Collaboration, Eur.Phys.J.C68, 89 (2010); ibid C68, 345(2010), CMS Collaboration, JHEP 01, 079 (2011), ALICE Collaboration, Eur.Phys.J.C77, 33 (2017)

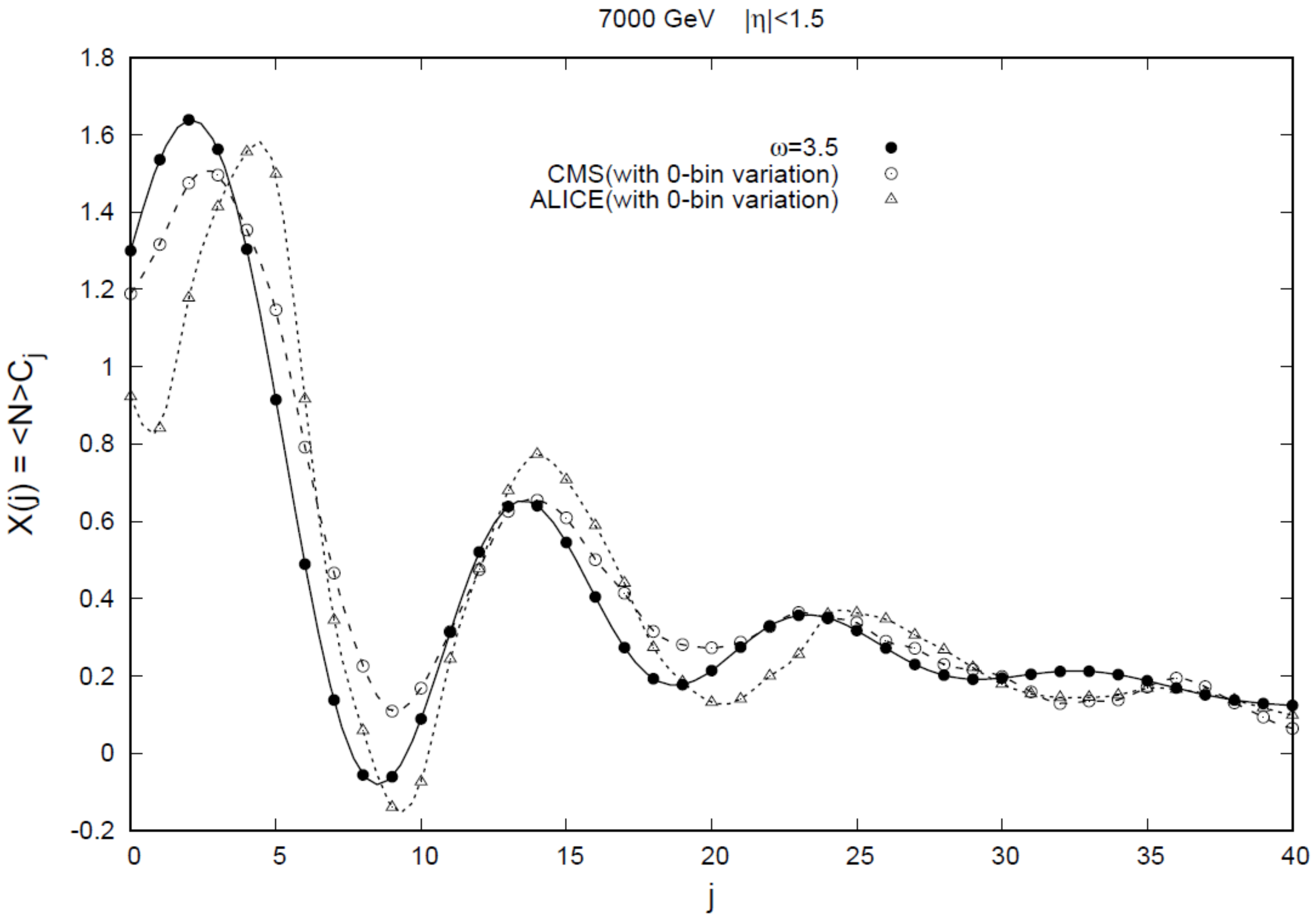
The problem of N=0 bin



At least one cut pomeron, n>0 =>
this excludes also the DD



Comparison to the pp experimental data with modified 0-bin



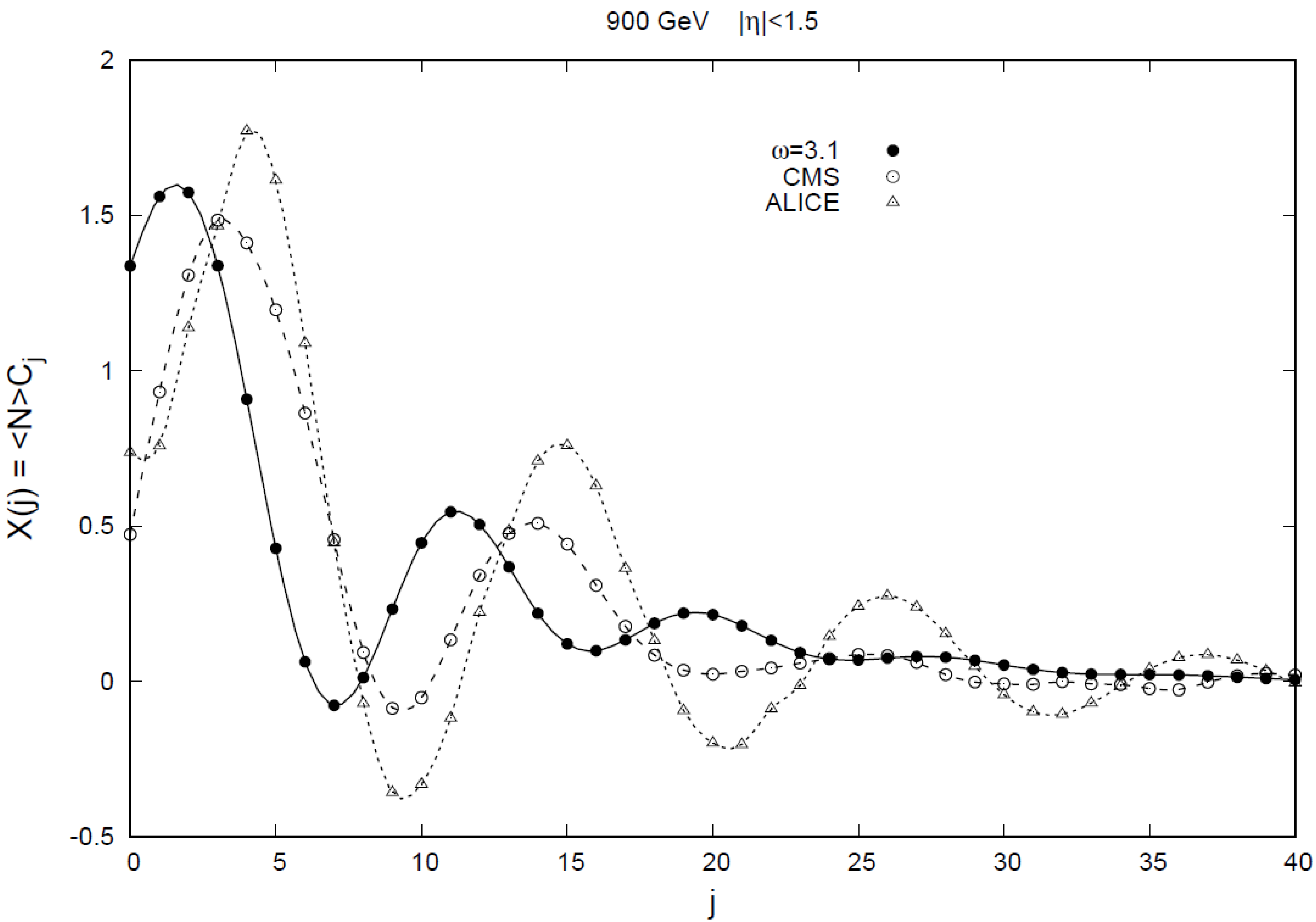
CMS Collaboration, JHEP 01, 079 (2011),
ALICE Collaboration, Eur.Phys.J.C77, 33 (2017)

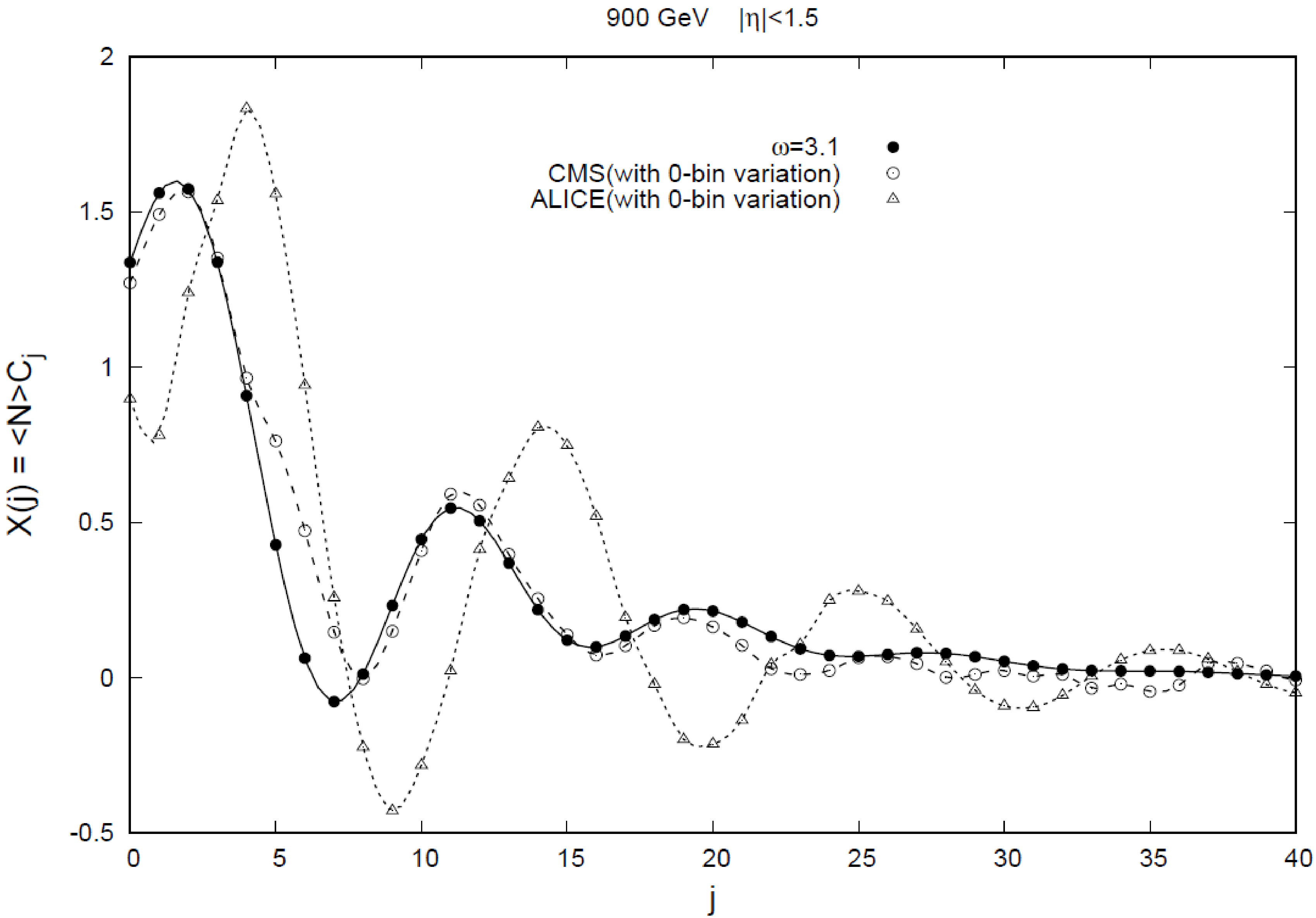
Concluding remarks

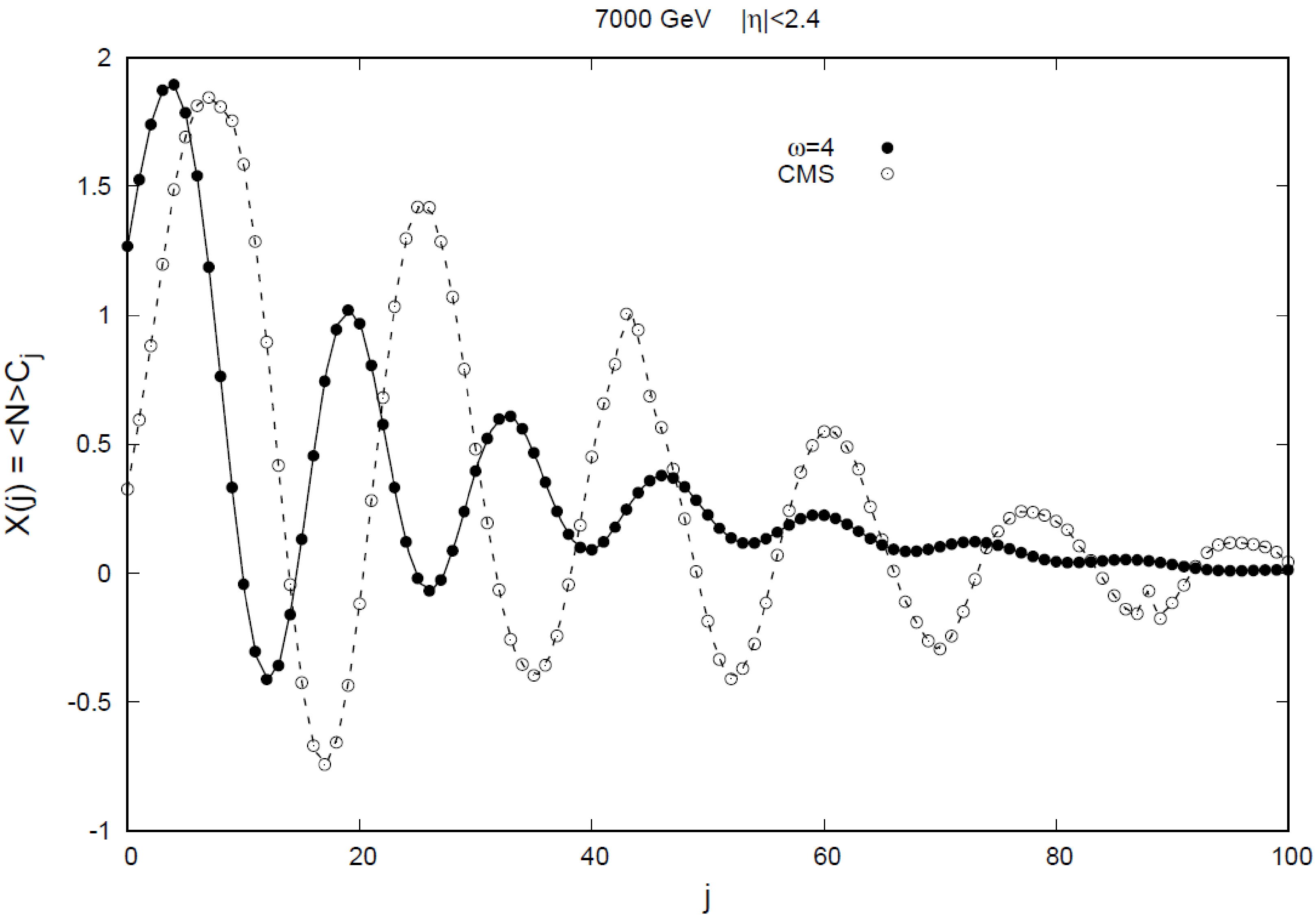
- The behavior of combinants proves to be a very sensitive tool in the analysis of particle multiplicity distribution. Even minor deviations of the ALICE and CMS data, within the error, lead to considerable changes in combinants.
- It is shown that within the framework of the multipomeron exchange model with Poisson or NBD distribution of particles from one source (quark-gluon string), it is not possible to explain the experimentally observed oscillations of combinants with increase of their number.
- Oscillations of combinants with increasing number can be explained if we assume a Gaussian form of particle multiplicity distribution at a fixed number of pomerons.
- The reason for the occurrence of these oscillations is the interplay between the contributions of 1, 2, 3, etc. cut pomerons.

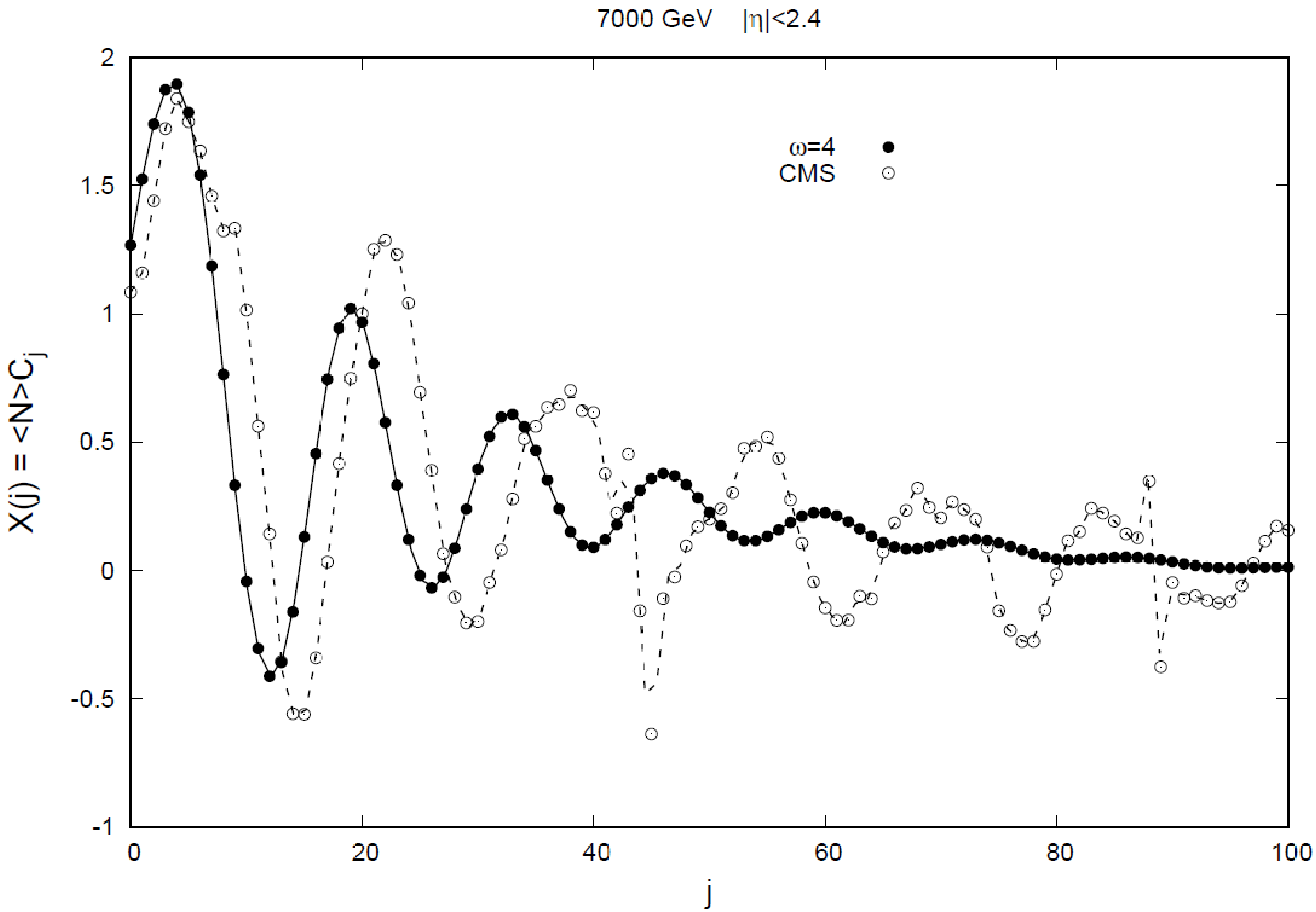
This research has been conducted with financial support from St.Petersburg State University (project No 93025435).

Backup slides

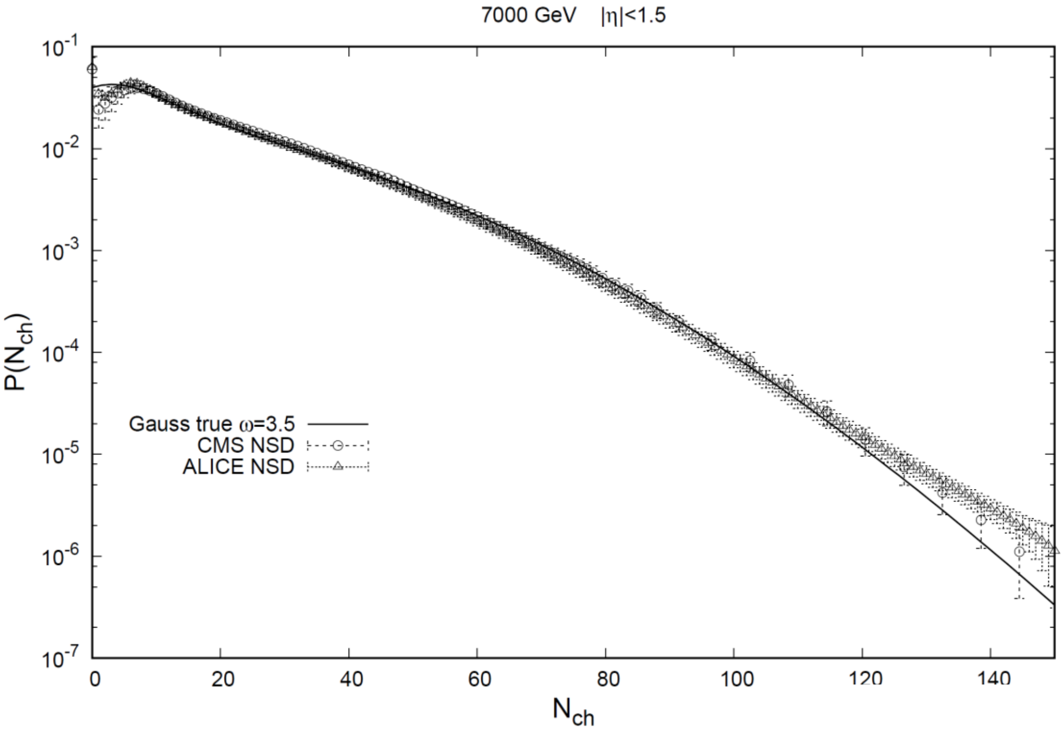

900 GeV |η|<1.5
ω=3.1
CMS
ALICE
X(j) = <N>C_j
j
2
1.5
1
0.5
0
-0.5
0
5
10
15
20
25
30
35
40


900 GeV |η|<1.5
ω=3.1
CMS(with 0-bin variation)
ALICE(with 0-bin variation)
X(j) = <N>C
j
2
1.5
1
0.5
0
-0.5
0
5
10
15
20
25
30
35
40
j

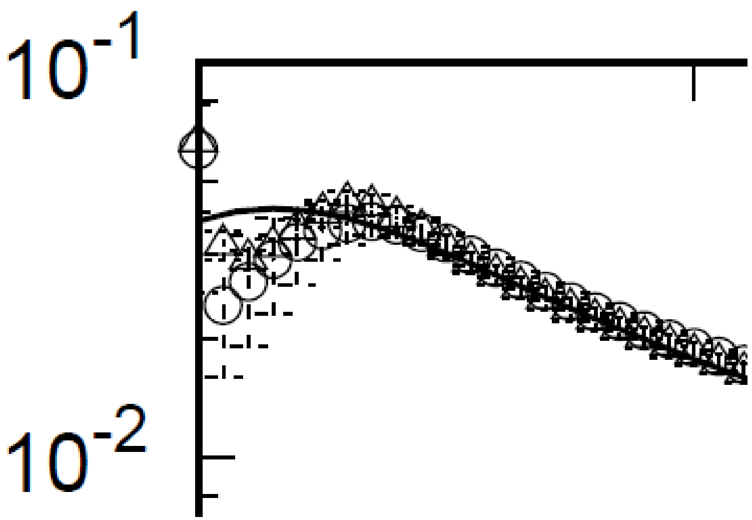

7000 GeV |η|<2.4
ω=4
CMS
X(j) = <N>C_j
j
2
1.5
1
0.5
0
-0.5
-1
0
20
40
60
80
100

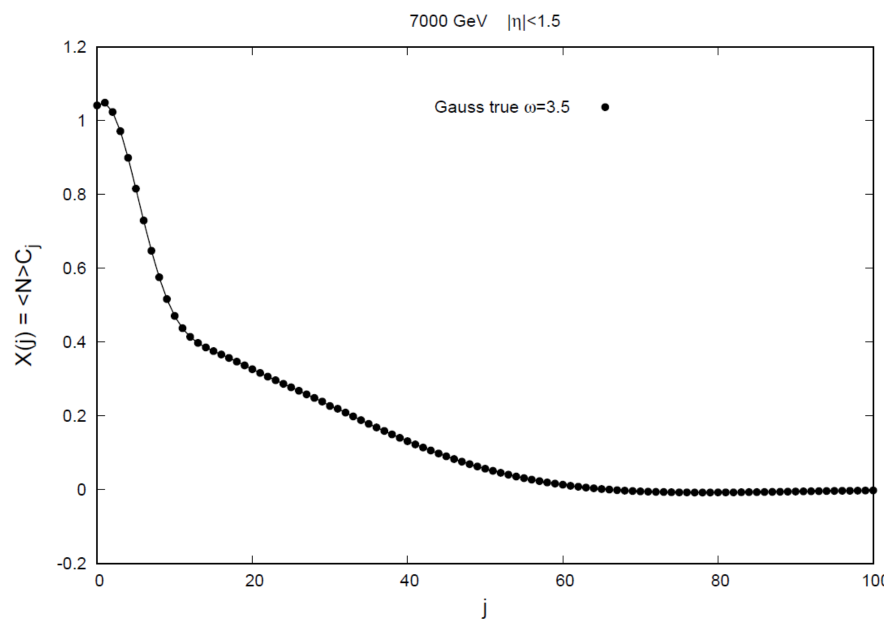

7000 GeV |η|<2.4
ω=4
CMS
X(j) = <N>C_j
j
2
1.5
1
0.5
0
-0.5
-1
0
20
40
60
80
100

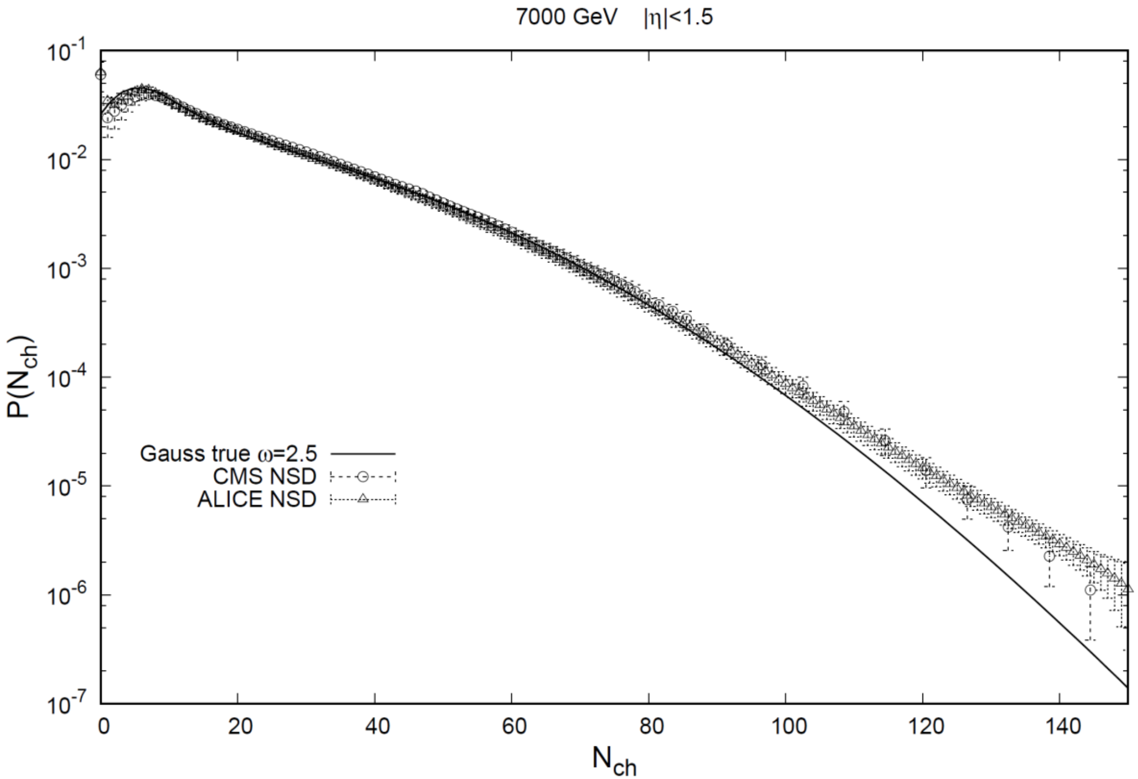
Assuming a Gaussian distribution for $P_n(N)$ with fixed $\omega_n = \omega$



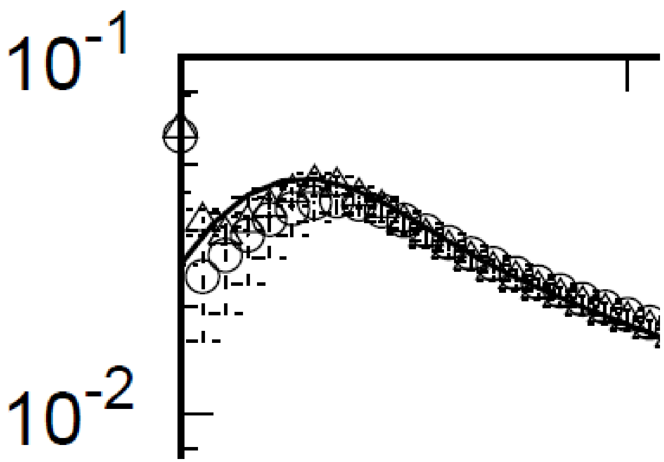
$\omega_n = 3.5$

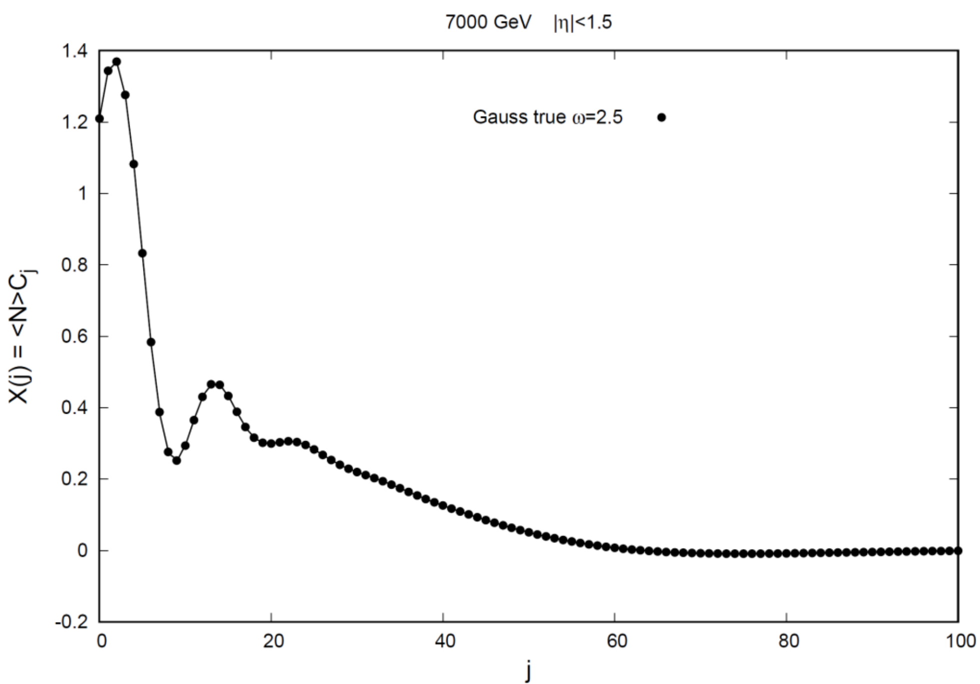


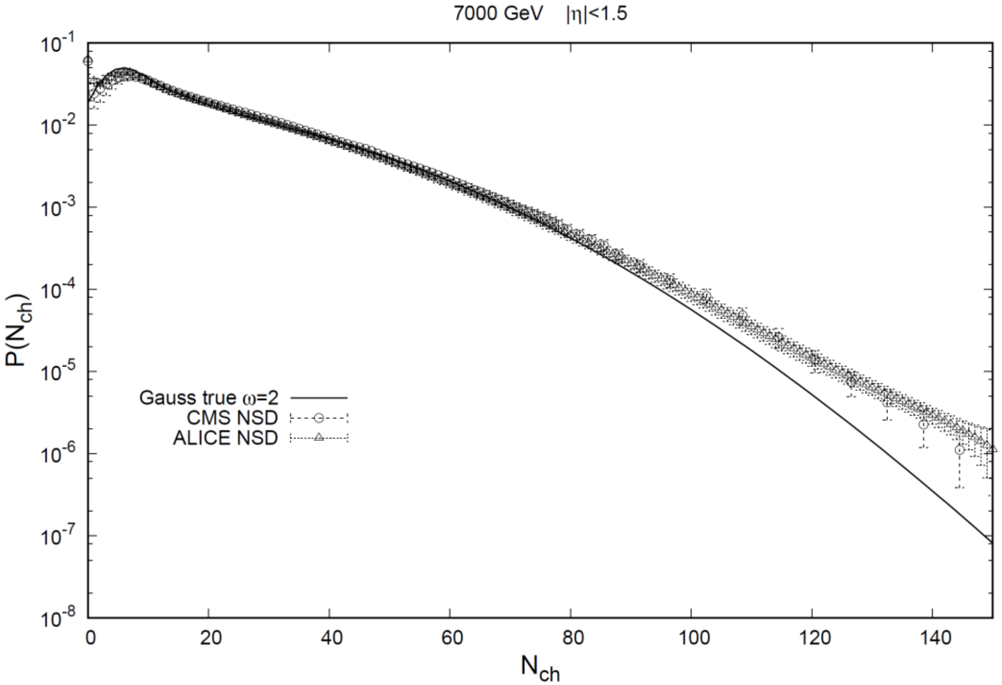



7000 GeV |η|<1.5
P(N_ch)
N_ch
Gauss true ω=2.5
CMS NSD
ALICE NSD

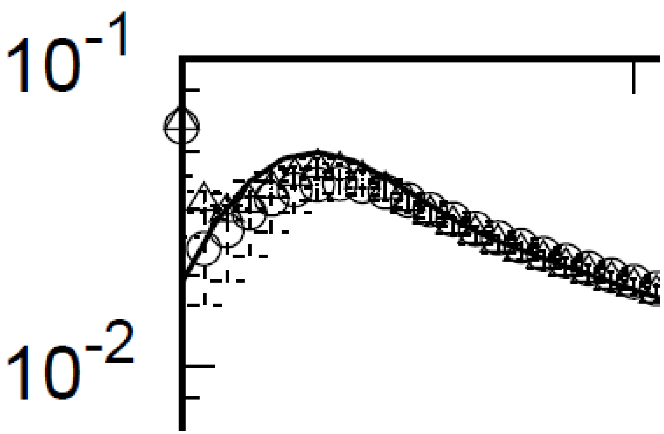
$$\omega_n = 2.5$$

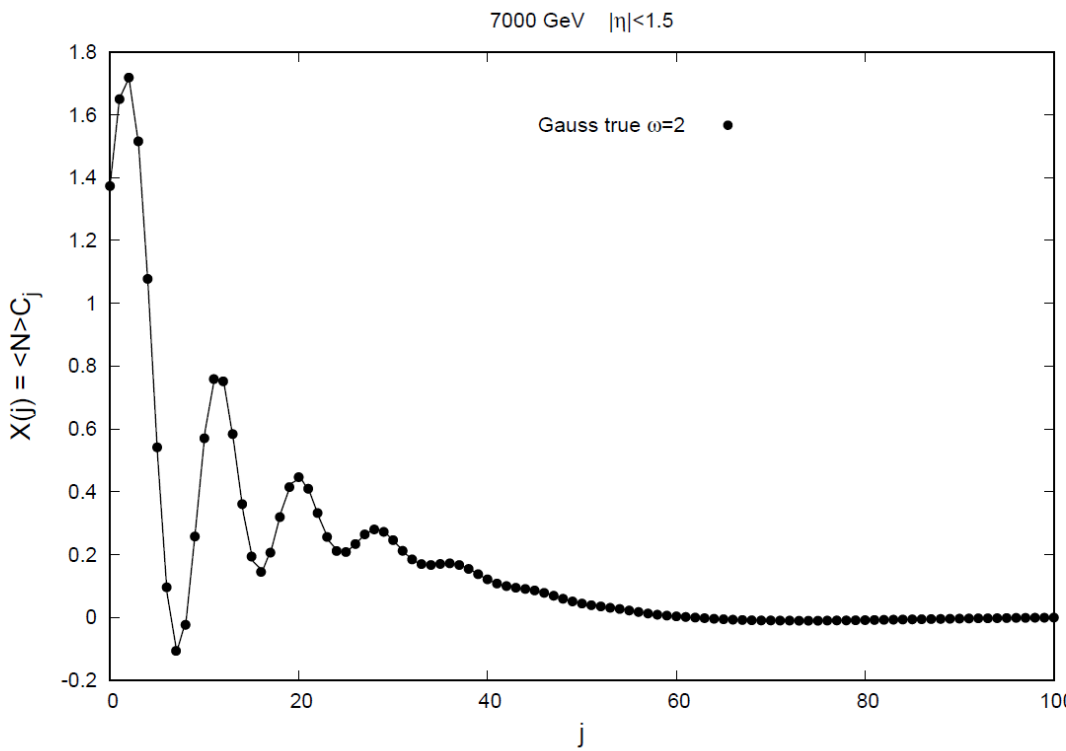



7000 GeV |η|<1.5
X(j) = <N>C_j
j
Gauss true ω=2.5


7000 GeV |η|<1.5
P(N_ch)
N_ch
Gauss true ω=2
CMS NSD
ALICE NSD

$$\omega_n = 2$$


10^-1
10^-2


7000 GeV |η|<1.5
X(j) = <N>C_j
j
Gauss true ω=2