Multiplicity distributions and combinatorics in multi-pomeron exchange model

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Multipliclity distribution and combinants

- Generating function for multiplicity distribution:

\[ G(t) = \sum_{N=0}^{\infty} P(N) \, t^N \]

- Generating function for combinants:

\[ F(t) = \sum_{j=0}^{\infty} C^*(j) \, t^j, \quad \text{where} \quad F(t) = \ln G(t) \]

- Recurrence relation for combinants:

\[ N \, P(N) = \sum_{j=1}^{N} j \, C^*(j) \, P(N - j) \]
Modified combinants

\[ C(j) \equiv \frac{j+1}{\langle N \rangle} C^*(j+1), \quad \text{where} \quad \langle N \rangle = \sum_{N=1}^{\infty} N P(N) \]

- Recurrence relation for modified combinants:

\[ (N + 1) P(N + 1) = \langle N \rangle \sum_{j=0}^{N} C(j) P(N - j) \]

Relationships of this type or even simpler ones:

\[ (N + 1) P(N + 1) = g(N) P(N) \]

where \( g(N) \) is a linear function, are found in many models (clans, cascade models).

See e.g. formula (32) in


\[ X(j) \equiv \langle N \rangle C(j) = (j + 1) C^*(j + 1) \]
Using the recurrence relation to calculate combinants

\[ X(N) = (N + 1) \frac{P(N + 1)}{P(0)} - \sum_{j=0}^{N-1} X(j) \frac{P(N - j)}{P(0)} \]

Explicit solution for the first few

\[ X(j) \equiv \langle N \rangle C(j) = (j + 1) C^*(j + 1) \]

\[ \overline{P}(N) \equiv \frac{P(N)}{P(0)} = \frac{P(1)}{P(0)} \]

\[ X(0) = \overline{P}(1) = \frac{P(1)}{P(0)} \]

\[ X(1) = 2\overline{P}(2) - \overline{P}^2(1) = 2 \frac{P(2)}{P(0)} - \left( \frac{P(1)}{P(0)} \right)^2 \]

\[ X(2) = 3\overline{P}(3) - 3\overline{P}(1)\overline{P}(2) + \overline{P}^3(1) \]

\[ X(3) = 4\overline{P}(4) - 4\overline{P}(1)\overline{P}(3) - 2\overline{P}^2(2) + 4\overline{P}^2(1)\overline{P}(2) - \overline{P}^4(1) \]
Why combinants?

Multiplicity distributions in pp collisions at LHC energies

Multiplicity distributions in pp collisions at LHC energies

Combinants extracted from the experimental data


Multi-Pomeron Exchange Model


\[ P(N) = C(z) \sum_{N_{pom}} \frac{1}{z \cdot N_{pom}} \left( 1 - e^{-z} \sum_{l=0}^{N_{pom}-1} \frac{z^l}{l!} \right) \cdot P_{N_{pom}}(N) \]

\[ z = \frac{2C\gamma s^{\Delta}}{R^2 + \alpha' \log(s)}, \quad \Delta = 0.139, \quad \alpha' = 0.21 \text{ GeV}^{-2}, \quad \gamma = 1.77 \text{ GeV}^{-2}, \quad R_0^2 = 3.18 \text{ GeV}^{-2}, \quad C = 1.5 \]

\[ P_{N_{pom}}(N) \text{ was taken to be Poisson distribution with mean} \]
\[ 2 \cdot N_{pom} \cdot \delta \eta \cdot k(\sqrt{s}) \text{ where } k = 0.255 + 0.0653 \cdot \ln \sqrt{s} \]

Regge parameters are taken from:
Each cut pomeron corresponds to a pair of quark-gluon strings.
An increase in the multiplicity density with energy is explained by an increase in the mean number of cut pomerons and a growth of the average multiplicity from a single string.
Distribution in number of cut pomerons

Regge parameters are taken from:
Modeling a particle distribution from $n$ cut pomerons

$$P(N) = \sum_{N_{pom}=1}^{\infty} P(N_{pom}) P_{N_{pom}}(N) = \sum_{n=1}^{\infty} P(n) P_{n}(N), \quad n \equiv N_{pom}$$

$$\langle N\rangle_n = 2n \delta \eta k(s) = 2n \langle N \rangle_{str} \equiv 2n \mu_{str}$$

**Poisson Distribution**

$$P_{n}(N) = e^{-\langle N \rangle_n} \frac{\langle N \rangle_{n}^{n}}{N!}, \quad P_{str}(N) = e^{-\mu_{str}} \frac{\mu_{str}^{N}}{N!}$$

**Negative Binominal Distribution**

$$P_{n}(N) = \frac{p^{N}}{q^{N+k}} \frac{\Gamma(N+k)}{\Gamma(k) N!}, \quad g_{NBD}(t) = (q - p t)^{-\kappa}$$

$$q = \omega \equiv \frac{\langle N^2 \rangle_{n} - \langle N \rangle_{n}^2}{\langle N \rangle_{n}} = \frac{\langle N^2 \rangle_{str} - \langle N \rangle_{str}^2}{\langle N \rangle_{str}} \equiv \omega_{str} \quad \Rightarrow \quad \text{true also for merged, but identical strings}$$

$$p = q - 1 = \omega_{str} - 1, \quad \kappa = \frac{\langle N \rangle_{n}}{p} = 2n \frac{\mu_{str}}{\omega_{str} - 1}$$
Fluctuation in the number of particles from one string


In the presence of correlations between particles, there can be no Poisson distribution from the source.

\[
\frac{\langle N^2 \rangle_{str} - \langle N \rangle_{str}^2}{\langle N \rangle_{str}} \equiv \omega_{str} = 1 + \mu_{str} J, \quad J \equiv \frac{1}{\delta \eta^2} \int_{\delta \eta} d\eta_1 \int_{\delta \eta} d\eta_2 \Lambda(\eta_1 - \eta_2)
\]

The pair correlation function of a single string:

\[
\Lambda(\eta_1, \eta_2) \equiv \frac{\lambda_2(\eta_1, \eta_2)}{\lambda(\eta_1)\lambda(\eta_2)} - 1 = \frac{\lambda_2(\eta_1 - \eta_2)}{\mu_0^2} - 1 = \Lambda(\eta_1 - \eta_2)
\]

The one- and two-particle rapidity distributions of particles from one string decay:

\[
\lambda(\eta) \equiv \frac{dN}{d\eta} = \frac{\mu_{str}}{\delta \eta} = \mu_0 = k(s), \quad \lambda_2(\eta_1, \eta_2) \equiv \frac{d^2N}{d\eta_1 d\eta_2} = \lambda_2(\eta_1 - \eta_2).
\]

The parametrization for the pair correlation function of a single string \( \Lambda(\Delta \eta, \Delta \phi) \)

using the data from ALICE Collaboration, JHEP 05, 097 (2015),  
Forward-backward multiplicity correlations in pp collisions at 0.9, 2.76 and 7 TeV

Then integrating over azimuth we find:

\[
\Lambda(\Delta \eta) = \frac{1}{\pi} \int_0^{\pi} \Lambda(\Delta \eta, \Delta \phi) d\Delta \phi \quad \text{well approximated by} \quad \Lambda(\Delta \eta) = \Lambda_0 e^{-|\Delta \eta|/n_{corr}}
\]

V. Vechernin, EPJ Web Conf. 191, 04011 (2018),  
Fluctuation in the number of particles from one string

For the exponential pair correlation function, the integral can be calculated explicitly:

S. Belokurova, Phys. Part. Nucl. 53, 154 (2022)

\[ \omega_{\text{str}} = 1 + \mu_{\text{str}} J, \quad J = \frac{2\Lambda_0 \eta_{\text{corr}}}{(\delta\eta)^2} \left[ \delta\eta - \eta_{\text{corr}} \left( 1 - e^{-\delta\eta/\eta_{\text{corr}}} \right) \right] \]

Then we find:

\[ \frac{\langle N^2 \rangle_{\text{str}} - \langle N \rangle_{\text{str}}^2}{\langle N \rangle_{\text{str}}} \equiv \omega_{\text{str}} \]

<table>
<thead>
<tr>
<th>( \sqrt{s} ), TeV</th>
<th>0.9</th>
<th>7.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \delta\eta = 3 )</td>
<td>3.1</td>
<td>3.5</td>
</tr>
<tr>
<td>( \delta\eta = 4.8 )</td>
<td>3.6</td>
<td>4.0</td>
</tr>
</tbody>
</table>

Recall that in the multipomeron exchange model:

\[ \mu_{\text{str}} = k(s) \delta\eta \]
Assuming a Poisson distribution of particles from one string.
Assuming a Negative Binomial distribution from one string
Assuming a Gaussian distribution for $P_n(N)$

$$P_n(N) = C \exp \left[ -\frac{(N - 2n\mu_{str})^2}{2\omega_{str}^2 2n\mu_{str}} \right], \quad \sum_{N=0}^{\infty} P_n(N) = 1$$

$$C^{-1} = \sum_{N=0}^{\infty} \exp \left[ -\frac{(N - 2n\mu_{str})^2}{2\omega_{str}^2 2n\mu_{str}} \right]$$

For $2n\mu_{str} \gg 1$ we have

$$\langle N \rangle_n \equiv \sum_{N=1}^{\infty} N P_n(N) \to 2n\mu_{str}$$

$$\omega_n[N] \equiv \frac{\langle N^2 \rangle_n - \langle N \rangle_n^2}{\langle N \rangle_n} \to \omega_{str}$$
Comparison to the pp experimental data

CMS Collaboration, JHEP 01, 079 (2011),
CMS Collaboration, JHEP 01, 079 (2011),
The problem of N=0 bin

At least one cut pomeron, n>0 => this excludes also the DD
Comparison to the pp experimental data with modified 0-bin

CMS Collaboration, JHEP 01, 079 (2011),
Concluding remarks

• The behavior of combinatorials proves to be a very sensitive tool in the analysis of particle multiplicity distribution. Even minor deviations of the ALICE and CMS data, within the error, lead to considerable changes in combinatorials.

• It is shown that within the framework of the multipomeron exchange model with Poisson or NBD distribution of particles from one source (quark-gluon string), it is not possible to explain the experimentally observed oscillations of combinatorials with increase of their number.

• Oscillations of combinatorials with increasing number can be explained if we assume a Gaussian form of particle multiplicity distribution at a fixed number of pomerons.

• The reason for the occurrence of these oscillations is the interplay between the contributions of 1, 2, 3, etc. cut pomerons.

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Backup slides
Assuming a Gaussian distribution for $P_n(N)$ with fixed $\omega_n = \omega$.

$\omega_n = 3.5$
\[ \omega_n = 2.5 \]
\[ \omega_n = 2 \]