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DIFFUSENESS OF NUCLEON DENSITY DISTRIBUTION AND DOUBLE-FOLDING NUCLEUS-NUCLEUS POTENTIAL

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Fundamental problems and applications

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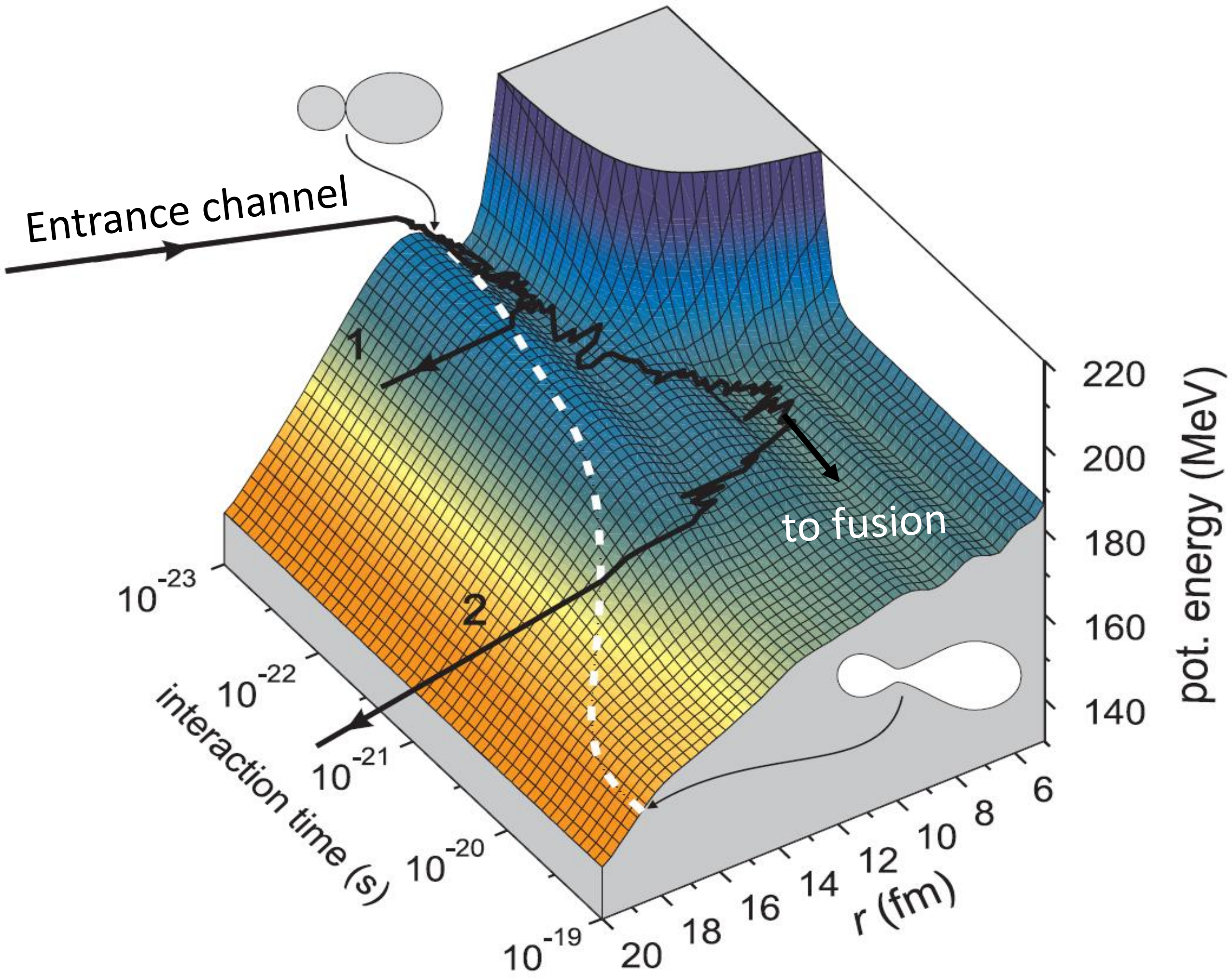
Low-energy reactions with heavy ions

Evolution of the system of colliding nuclei is determined by:

- nucleon density distribution in divided nuclei
- orientation and incident energy
- nucleon-nucleon interaction



1 – deep inelastic scattering
2 – quasi-fission

Low-energy reactions with heavy ions

Evolution of the system of colliding nuclei is determined by:

- nucleon density distribution in divided nuclei
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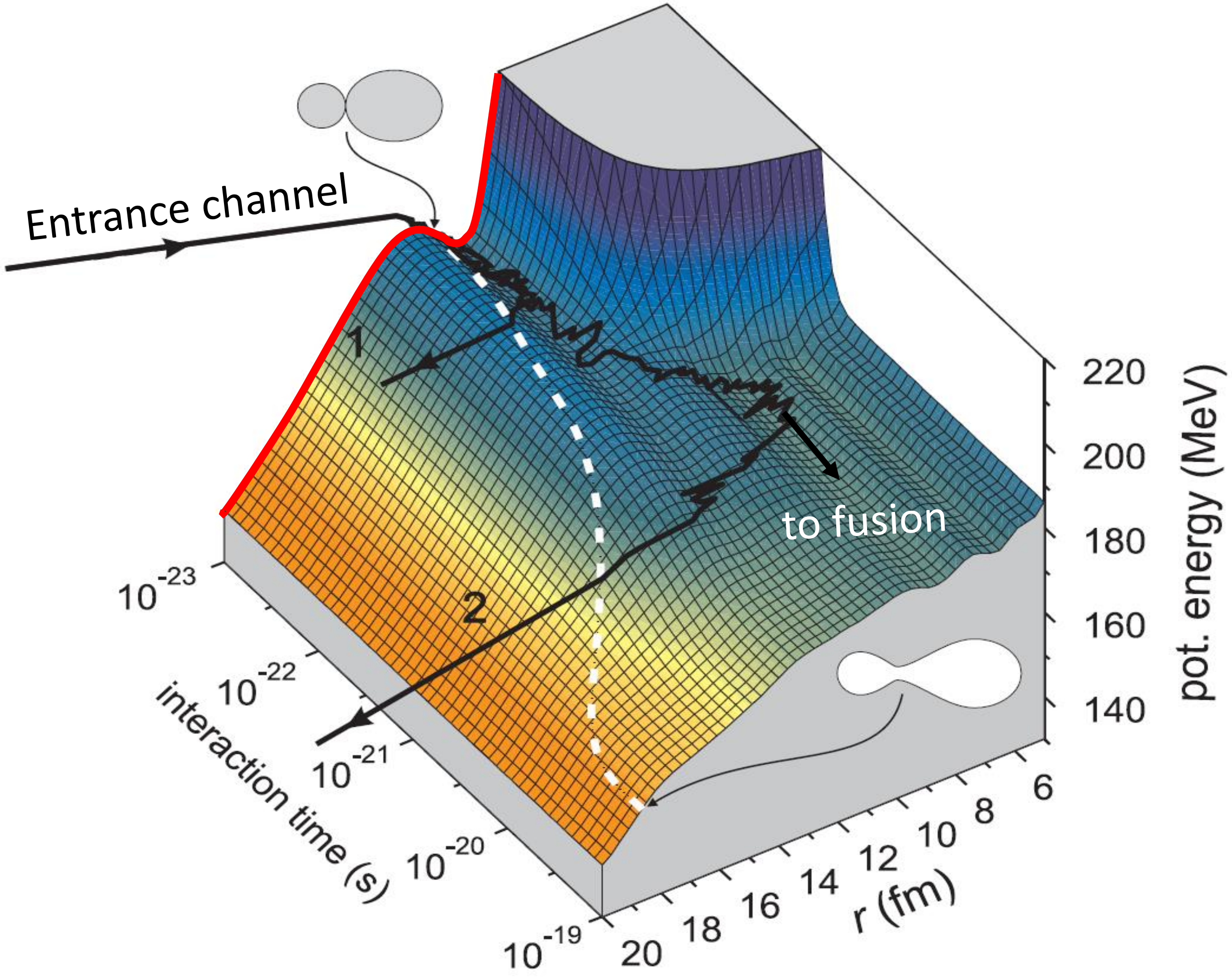
The key feature which is needed to evaluate probabilities of processes is barrier on the potential energy surface.



1 – deep inelastic scattering
2 – quasi-fission

Near-barrier reactions

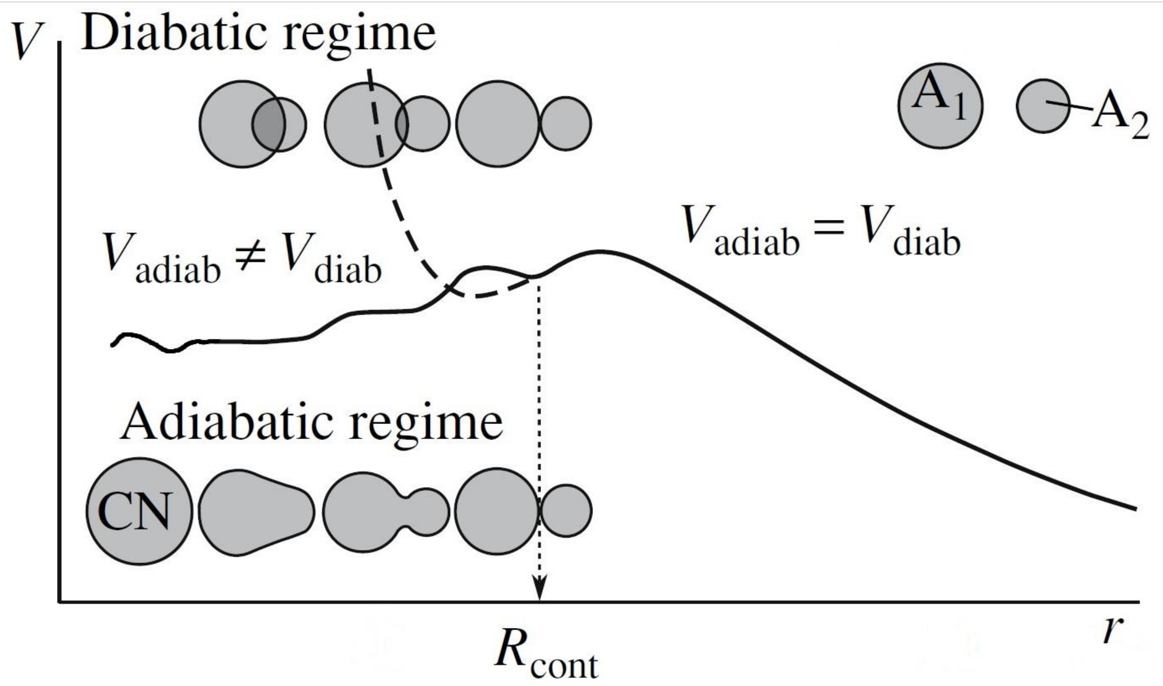
Two collision regime:

- diabatic (fast)
- adiabatic (slow)

Diabatic and adiabatic potential coincide before the *contact point*.

Objectives:

1. Investigate the impact of *nucleon density* parameters on the profile of diabatic potential
2. Calculate *diabatic potential* for spherical nuclei
3. Evaluate changes in *surface diffuseness* during near-barrier reaction



Adopted from: **Zagrebaev V. et al.**// Phys. Part. Nucl., 2007. 38, 4, 469–491

$$V_{diab}(r) = V_{NN}(r) + V_{Coulomb}(r)$$

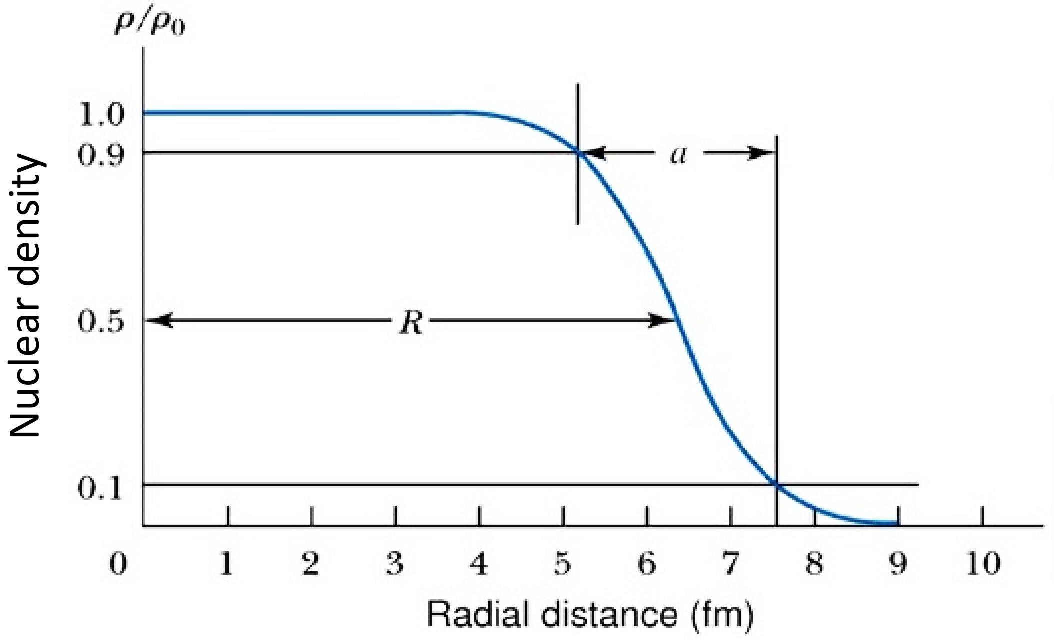
Double-folding potential

1. Double-folding potential: $$V_{NN}(r) = \int_{V_1} \rho_1(\boldsymbol{r}_1) \int_{V_2} \rho_2(\boldsymbol{r}_2)\, v_{NN}(\boldsymbol{r} + \boldsymbol{r}_2 - \boldsymbol{r}_1)\, \mathrm{d}^3 r_2\, \mathrm{d}^3 r_1$$

2. Migdal potential: $$v_{NN}(\boldsymbol{r}_{12}) = \mathrm{C}\left[\mathrm{F_{ex}} + (\mathrm{F_{in}} - \mathrm{F_{ex}})\frac{\rho_1(\boldsymbol{r}_1) + \rho_2(\boldsymbol{r}_2)}{\rho_2(0) + \rho_2(0)}\right]\delta(\boldsymbol{r}_{12})$$

3. Total nucleon density:

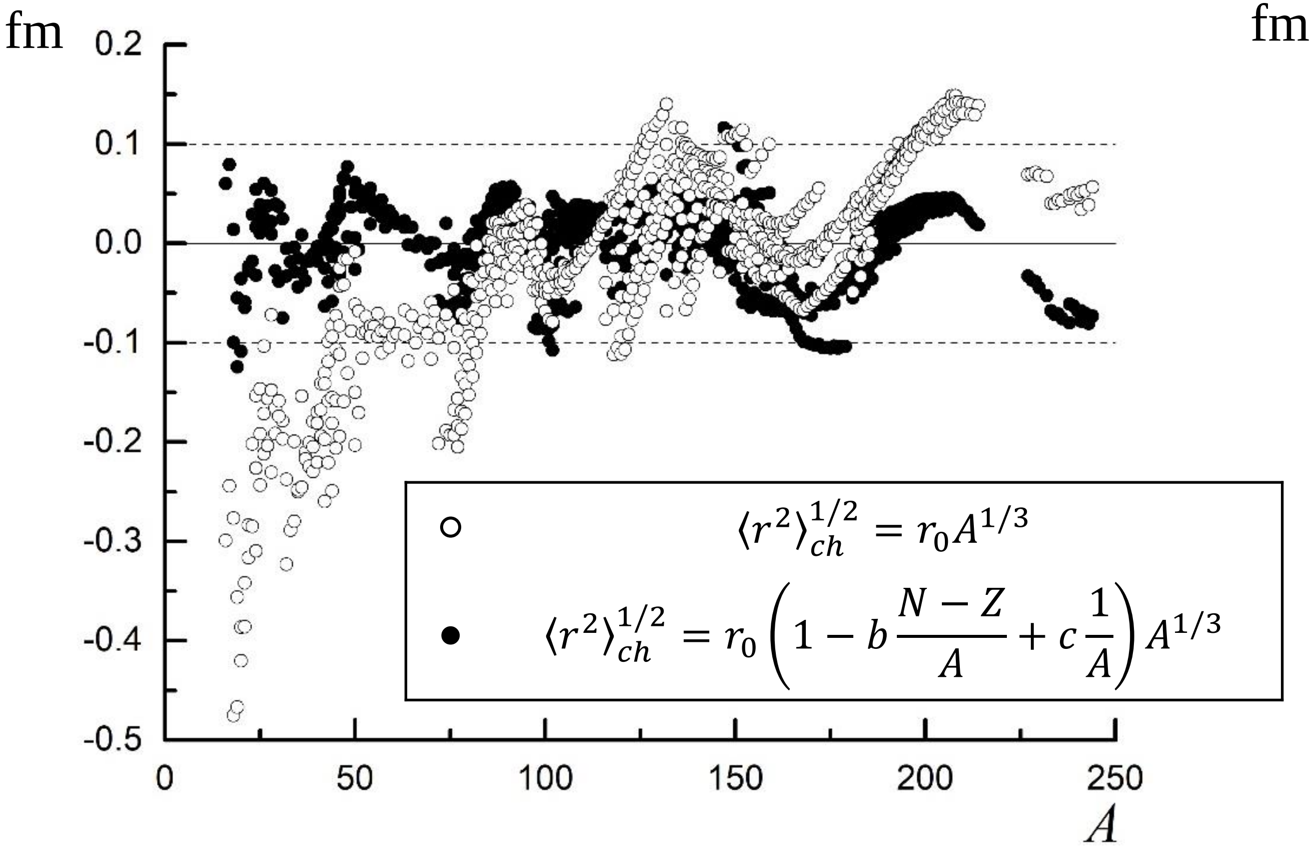
$$\rho_{1,2}(r) = \rho^{\mathrm{p}}_{1,2}(r) + \rho^{\mathrm{n}}_{1,2}(r),$$
$$\rho(r) = \frac{\rho_0}{1 + \exp\left(\frac{r - R_0}{a}\right)} \text{ for } p, n$$



4. Spherical nuclei $Z \geq 8$, $N \geq 8$

Charge density parameters

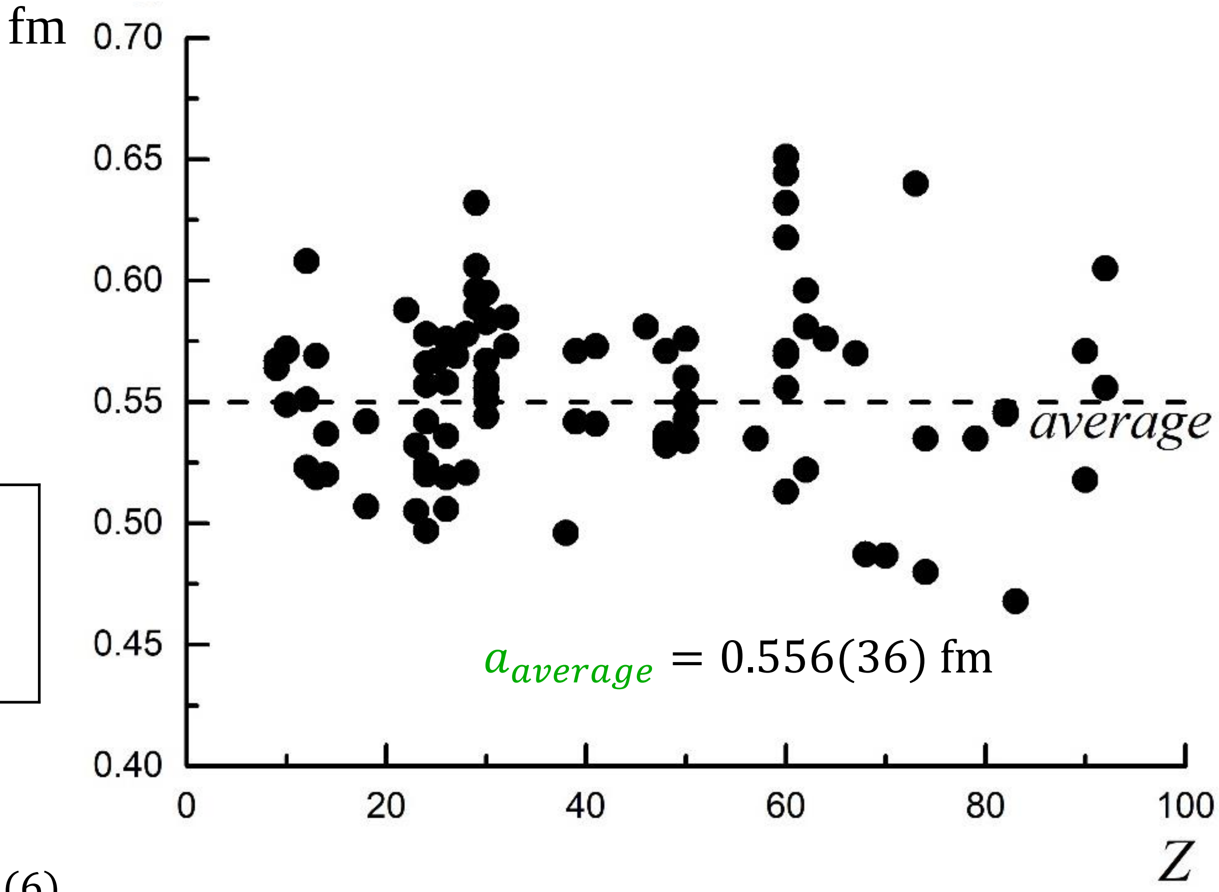
Residuals of RMS radii



New fit parameters: $r_0 = 0.956(14)$ fm, $b = 0.15(7)$, $c = 2.33(6)$

Exp. data: I. Angeli et al. ADNDT 99, 69 (2013).
Fit is proposed: B. Nerlo-Pomorska et al. Zeits. für Phys. A 348 (1994).

Diffuseness



Exp. data: H. de Vries et al. ADNDT 36, 495–536 (1987).

Neutron density parameters

Neutron density: $\rho_n(r) = \frac{\rho_{0n}}{1+\exp\left(\frac{r-R_n}{a_n}\right)}$

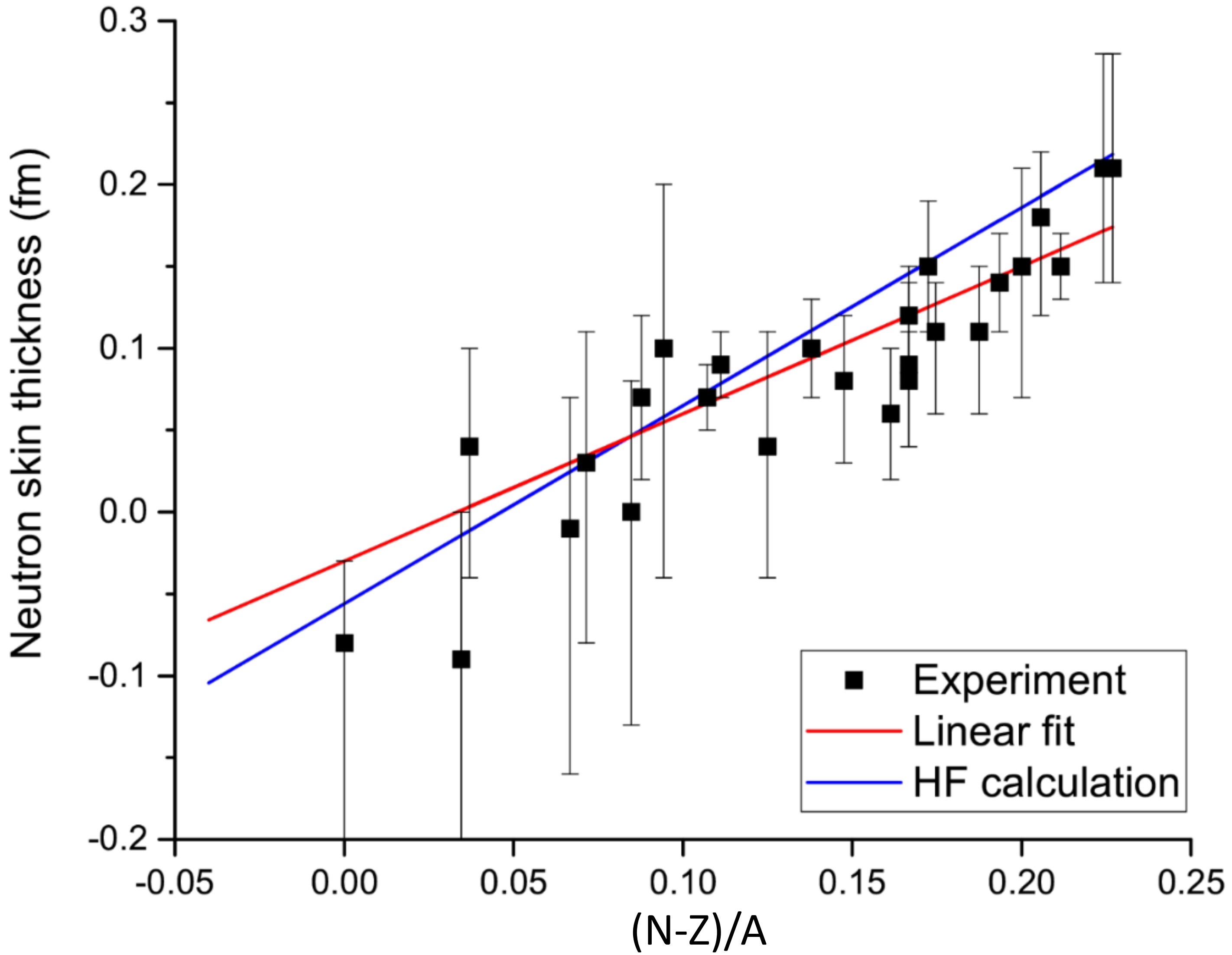
Neutron skin thickness:

$$\Delta R_{np} = \langle r^2 \rangle_n^{1/2} - \langle r^2 \rangle_p^{1/2}$$

Antiprotons-nuclei scattering
(approximation of experimental data):

$$\Delta R_{np} = a\frac{N-Z}{A} - b$$

$$a = 0.90(15)\ \mathrm{fm}$$
$$b = 0.03(2)\ \mathrm{fm}$$



***Exp. data and fit**: J. Jastrzebski et al. IJMP E, v. 13. (2004)*

Transformation of density parameters

The relation between point and matter/charge density:

$$\rho_{volume}(r) = \int f(\mathbf{r} - \mathbf{r}')\rho_{point}(\mathbf{r}')d\mathbf{r}'$$

Radii

Experimental data: $\langle r^2 \rangle_{ch}$

$$\langle r^2 \rangle_{ch}^{1/2} = r_0 \left(1 - b\frac{N-Z}{A} + c\frac{1}{A}\right) A^{1/3}$$

$\langle r^2 \rangle_{ch}$ from the fit

$$\langle r^2 \rangle_{ch} = \langle r^2 \rangle_p + \langle r^2 \rangle_{ch\_p} + \frac{N}{Z}\langle r^2 \rangle_{ch\_n}$$

$\langle r^2 \rangle_p$

$$R_{0p} = \sqrt{\frac{5}{3}\langle r^2 \rangle_p - \frac{7}{3}(\pi a_p)^2}$$

R_{0p}

$$\Delta R_{np} = a\frac{N-Z}{A} - b$$

Experimental data: ΔR_{np}

$\langle r^2 \rangle_n$

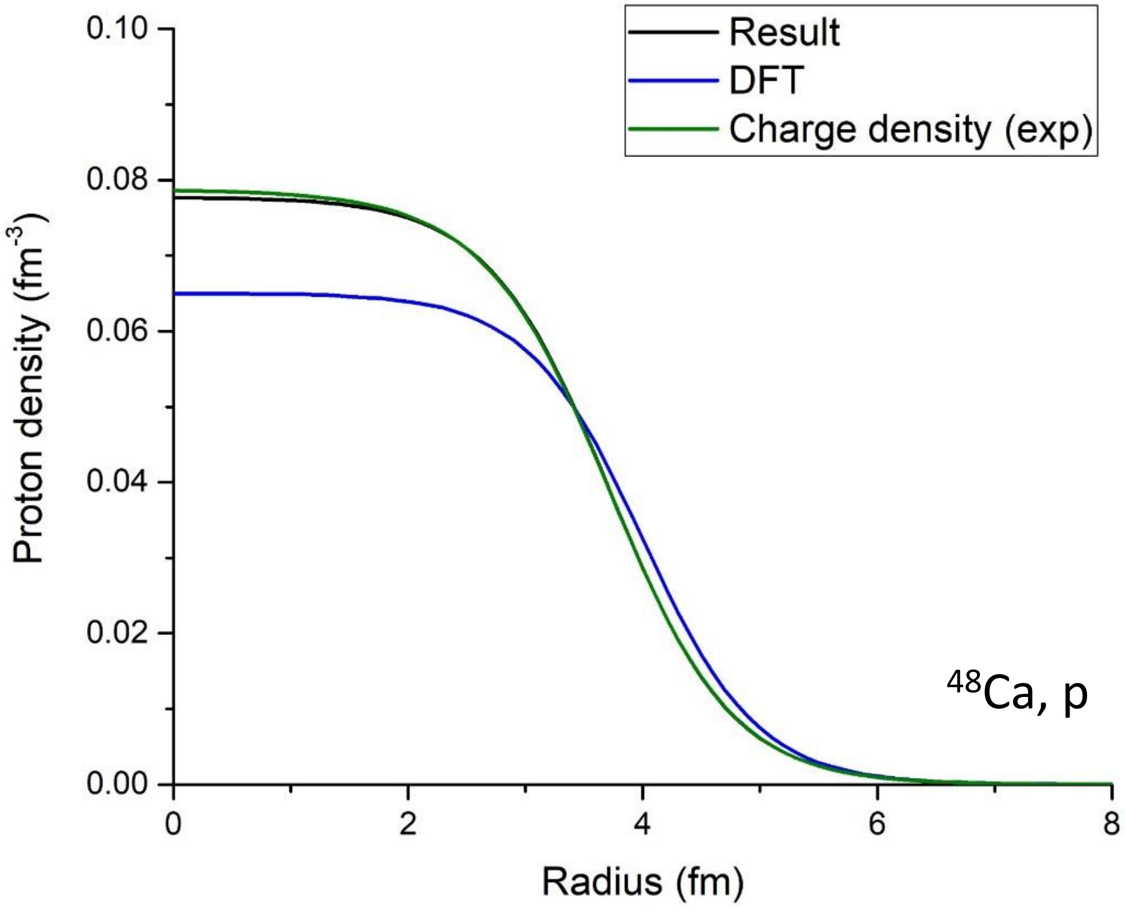
$$R_{0n} = \sqrt{\frac{5}{3}\langle r^2 \rangle_n - \frac{7}{3}(\pi a_n)^2}$$

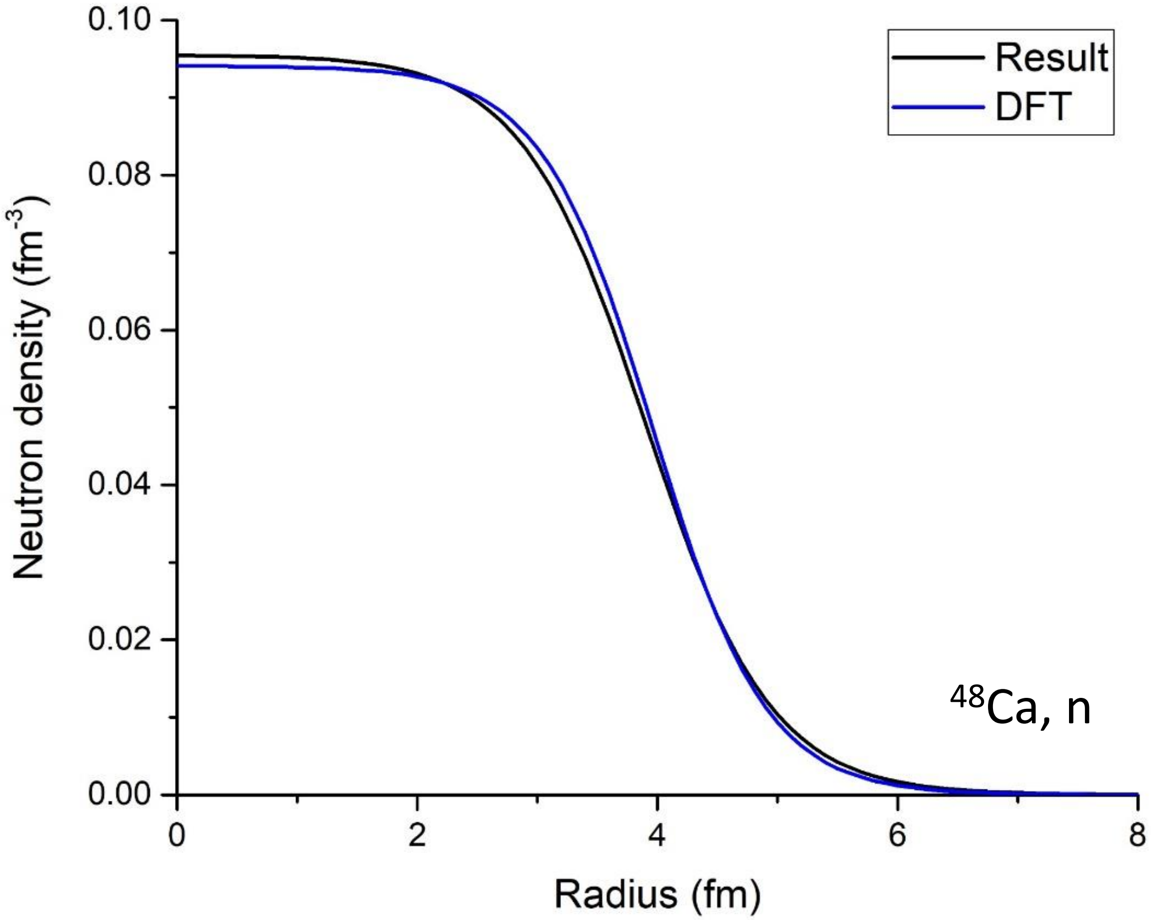
R_{0n}

Diffuseness $a_p = a_n = a_{ch} - 0.03 = 0.52$ fm

Approximation: Lima G. et al. Nucl. Phys A 735(3-4), 303 (2004).

An example of density calculation

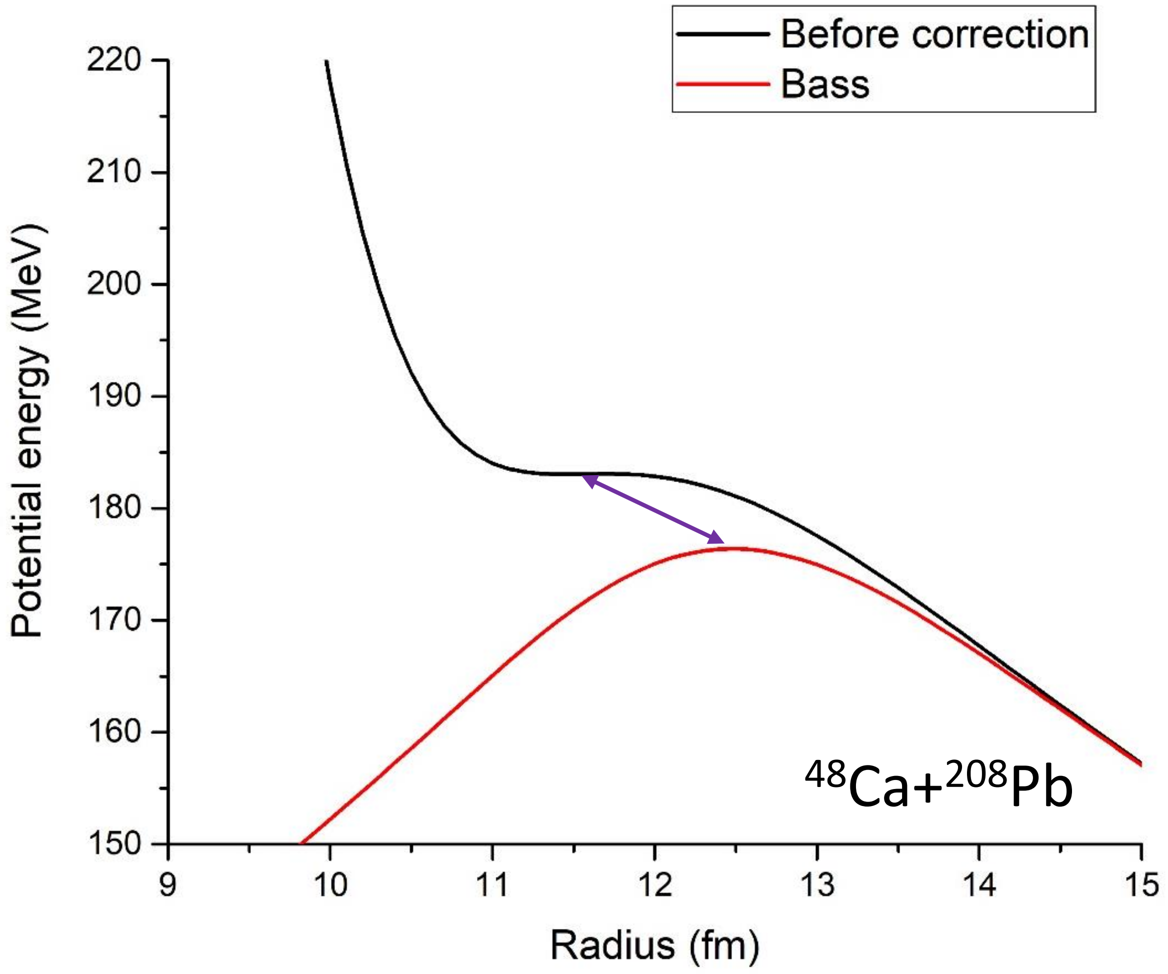


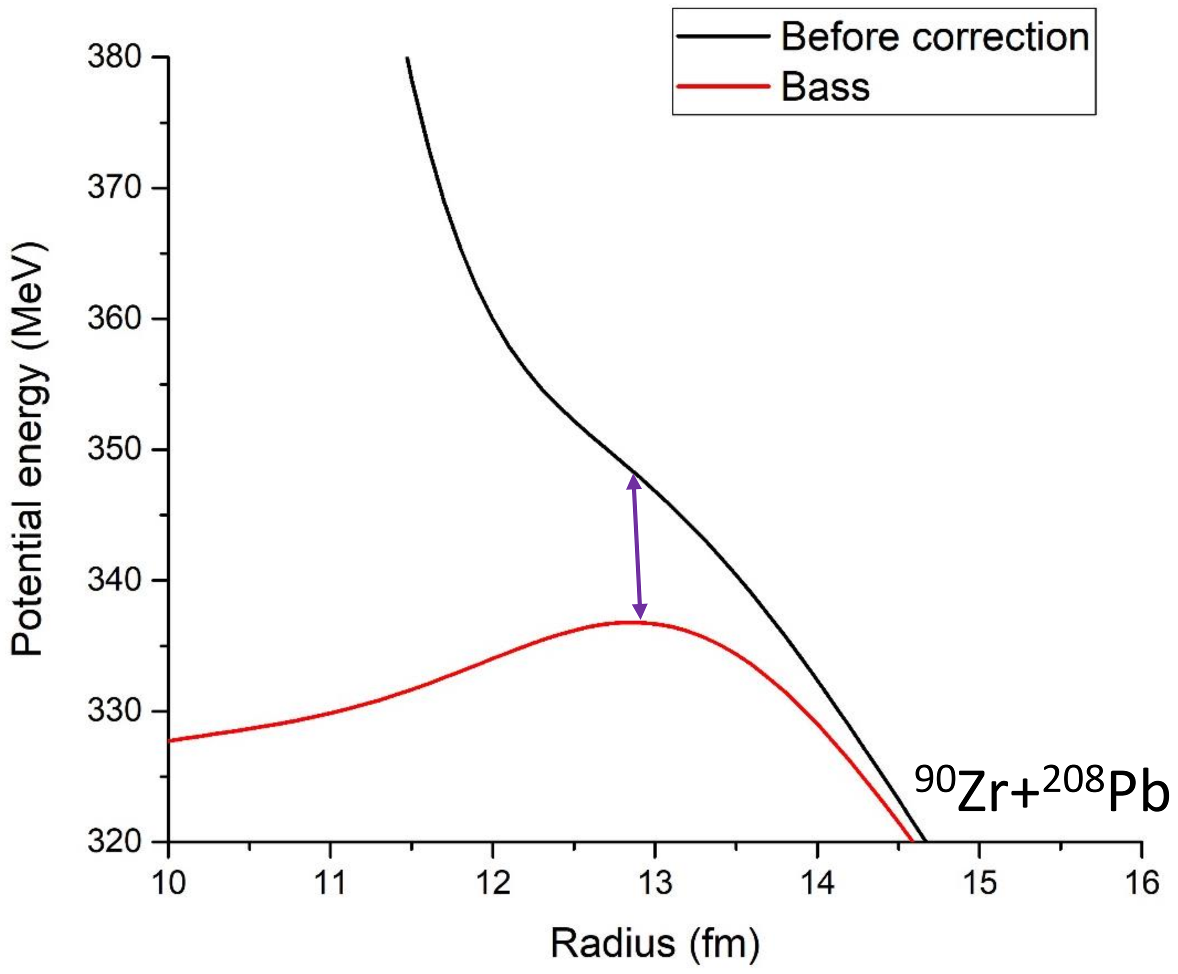


R_{0p}	R_{0n}	$\langle r^2 \rangle_p^{1/2}$	$\langle r^2 \rangle_{ch}^{1/2}$
3.72	3.91	3.47	3.55 (This work) 3.477(2)(Exp)

***Exp. data**: Bellicard J. B. et al. Phys. Rev. Lett. 19, (1967)*
***Theory (DFT)**: G. G. Adamian et al. Phys. Rev. C 94, 1 (2016).*

Diabatic potential



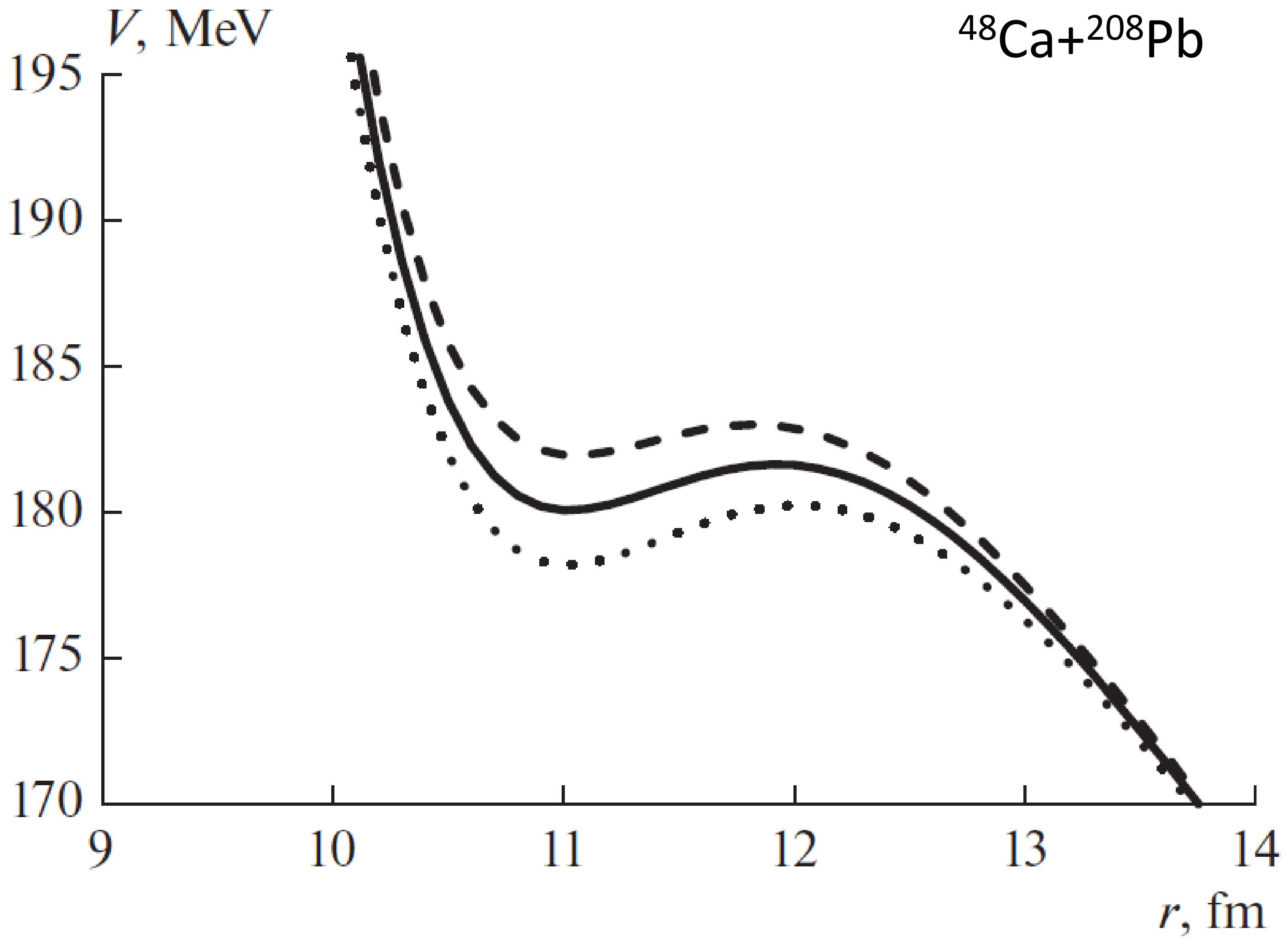


Bass potential: *R. Bass. Nucl. Reactions with Heavy Ions. Springer, 1980.*

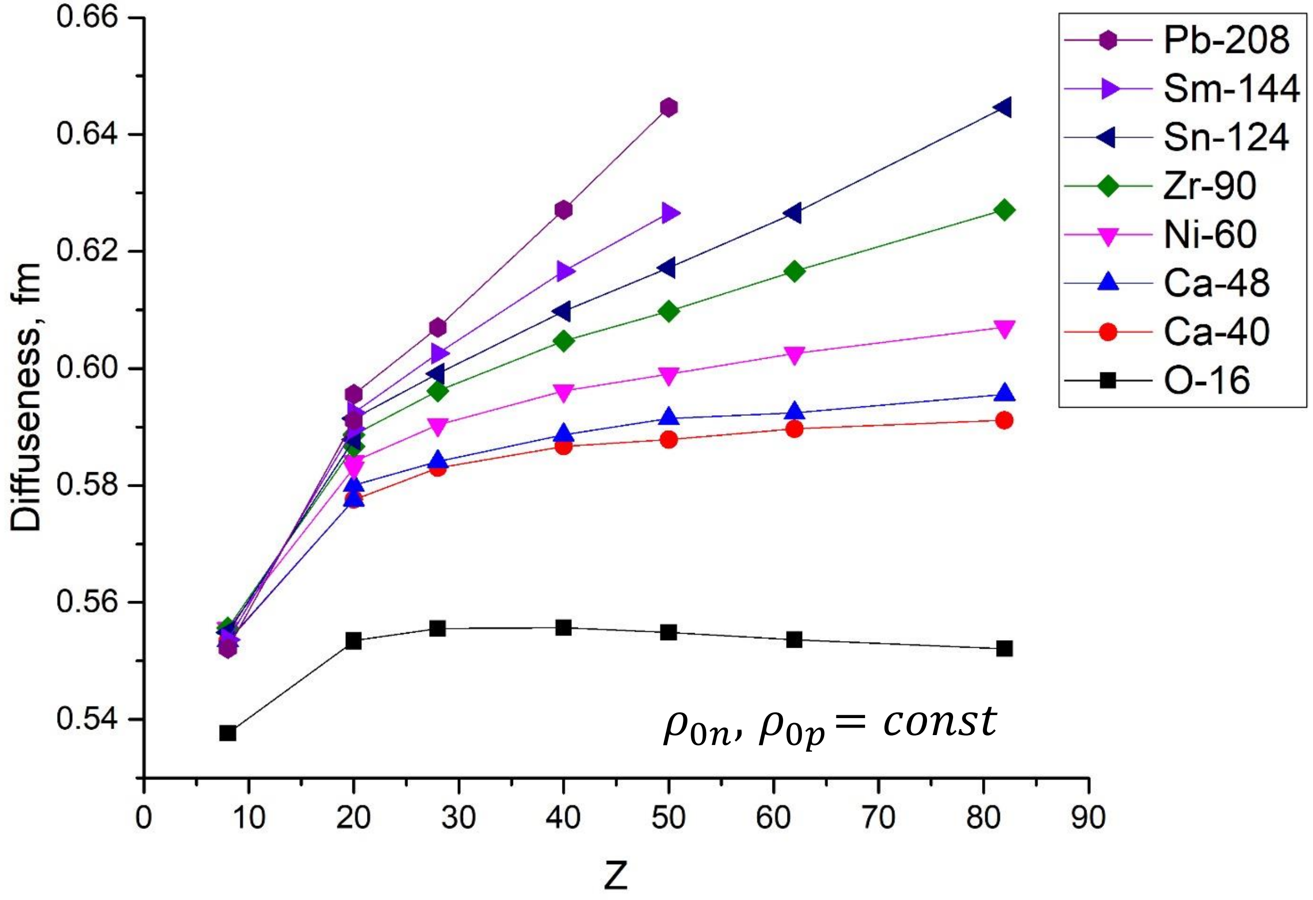
$$V_{\text{Bass}}(R) = \frac{Z_1 Z_2 e^2}{R} - \frac{R_1 R_2}{R_1 + R_2} g(R)$$

Variation of diffuseness

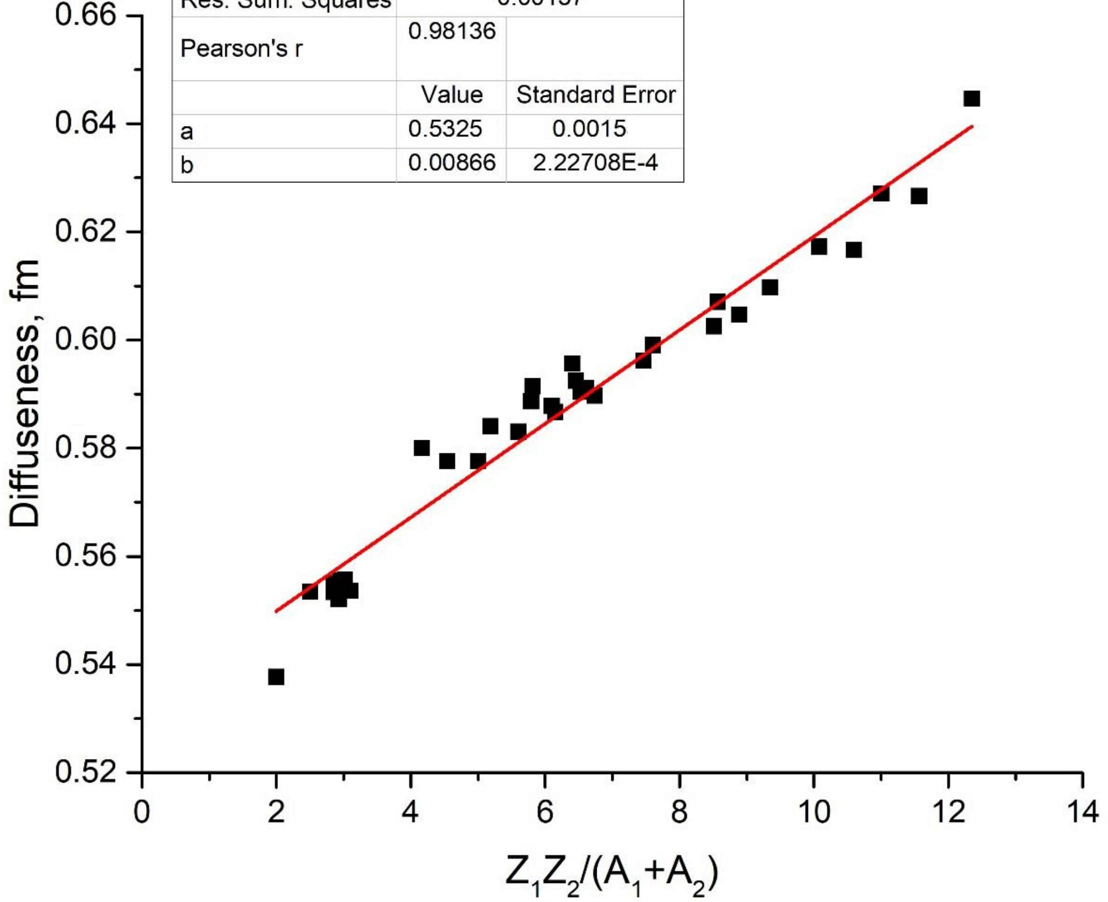
Diffuseness variation about 10 % for ^{48}Ca:
$a_{1p} = (0.55 \pm 0.06)$ fm



Correction of diffuseness



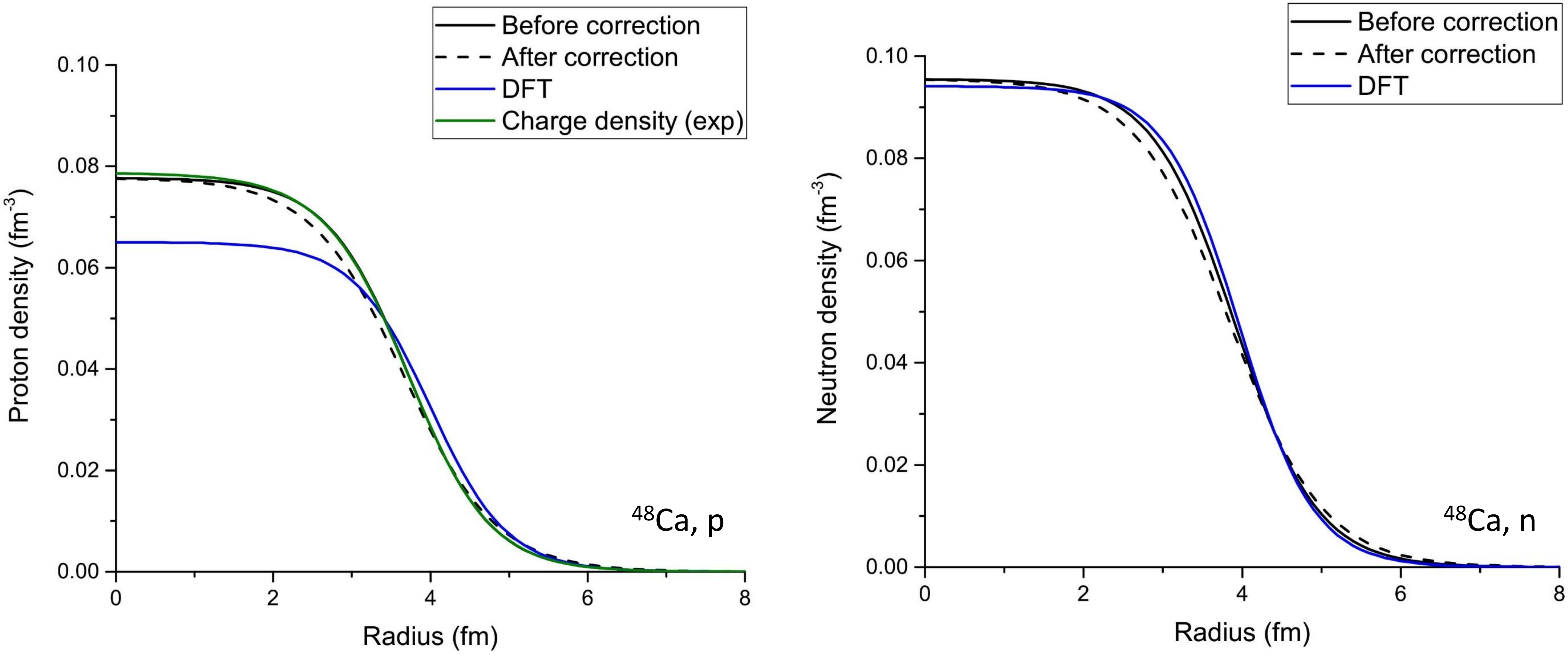
Diffuseness for different pairs of spherical nuclei



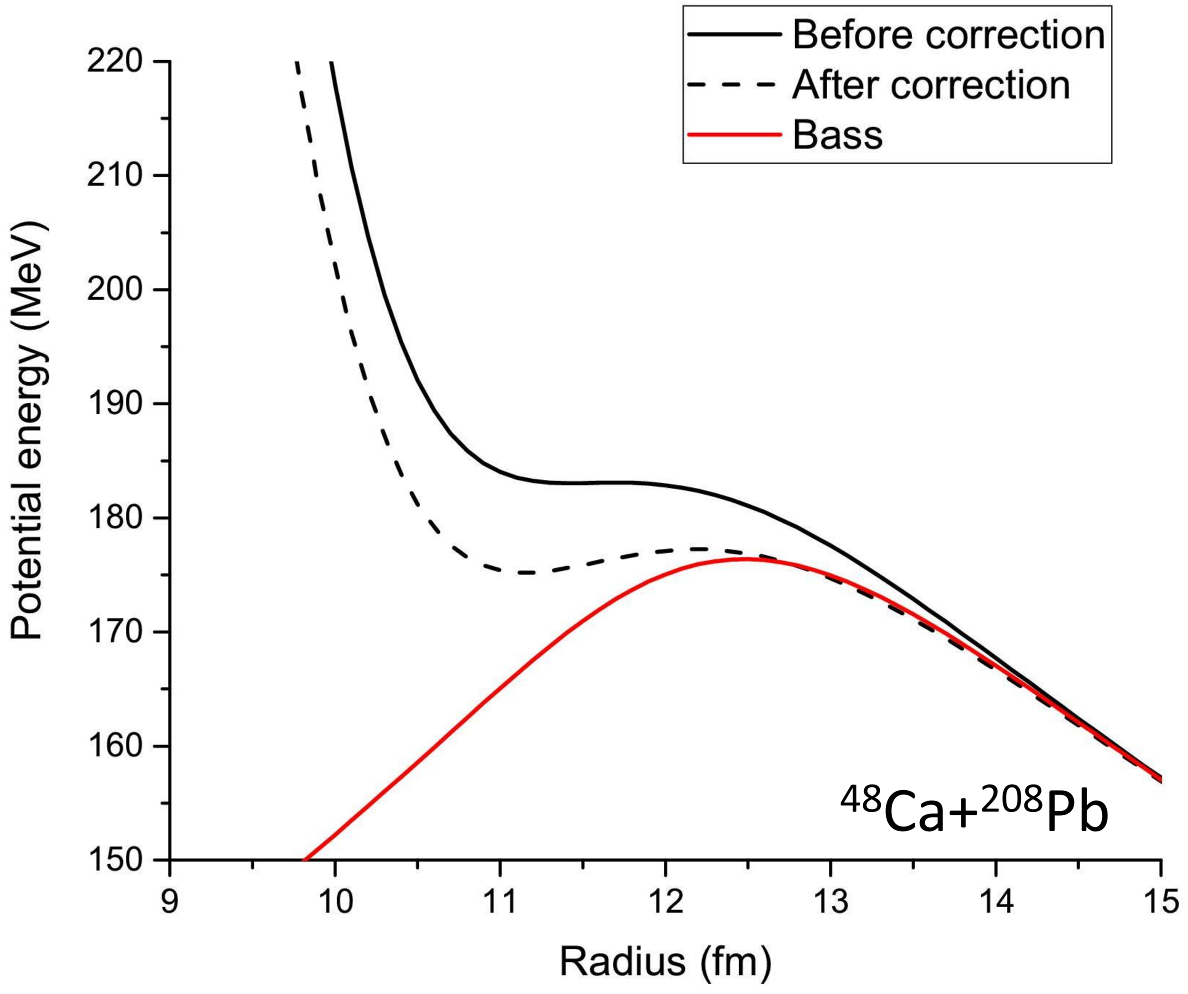
Equation	y = a + b*x	
Res. Sum. Squares	0.00137	
Pearson's r	0.98136	
	Value	Standard Error
a	0.5325	0.0015
b	0.00866	2.22708E-4

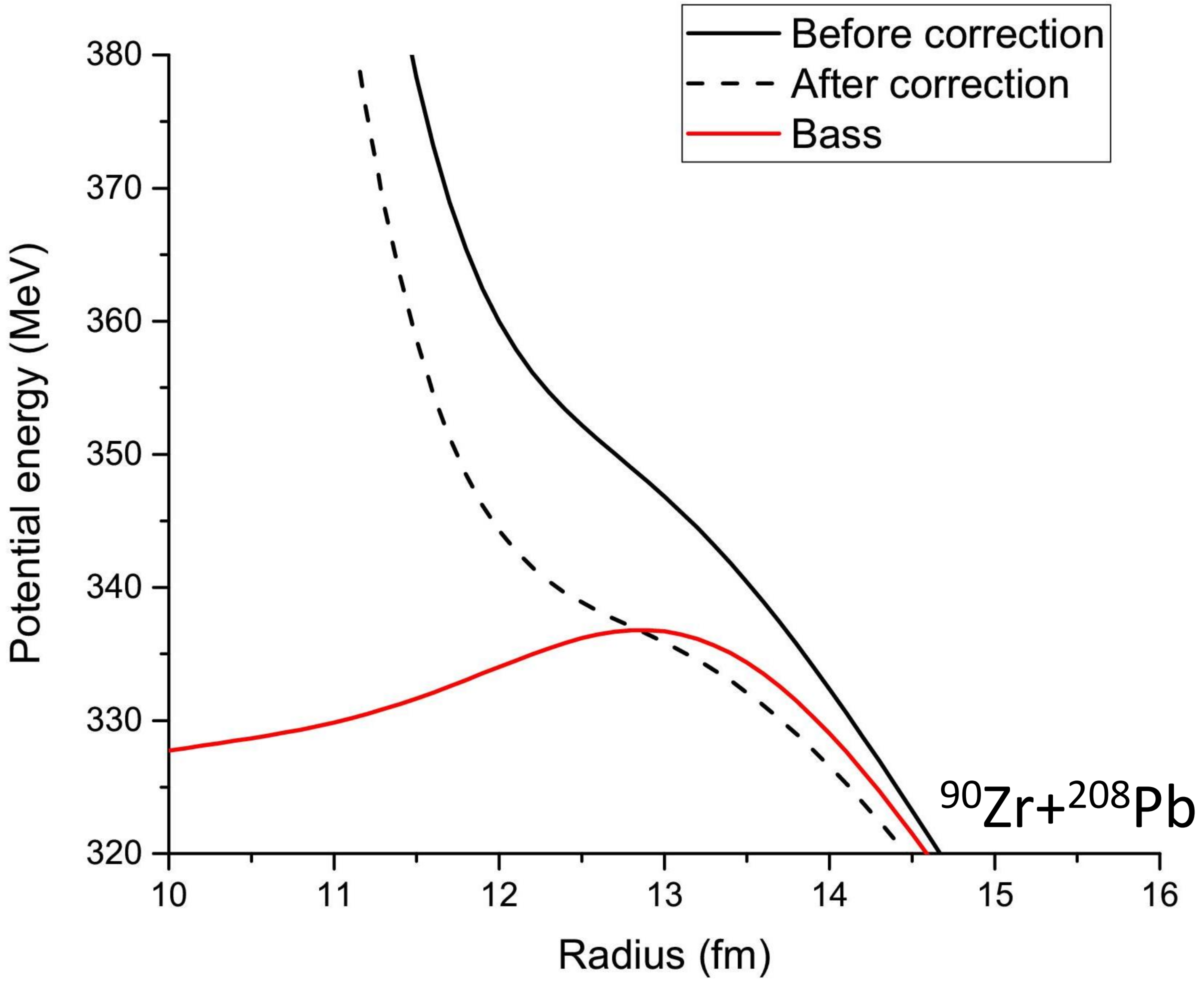
New linear fit: $a = a_0 + a_1 \frac{Z_1 Z_2}{A_1 + A_2}$

Corrected densities



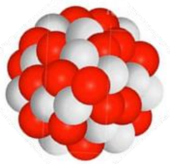
Corrected potentials





Conclusion

1. Experimental data on the **rms. charge radius**, the **charge diffuseness,** and the **neutron skin thickness** were analyzed.
2. Double-folding potential is calculated for spherical nuclei with $Z \geq 8$, $N \geq 8$ in the “frozen” density approximation.
3. It was found that the calculation carried out with the nucleon density for **isolated nuclei** leads to a potential that **differs significantly** from the Bass barrier.
4. For a more accurate assessment of the position of the Coulomb barrier, the **diffuseness** values were **corrected**. A **new approximation** for the diffuseness was proposed.



Thank you for your attention!

Migdal potential parameters

Effective nucleon-nucleon Migdal potential:

$$v_{NN}(\boldsymbol{r}_{12}) = \mathrm{C}\left[\mathrm{F}_{\mathrm{ex}} + (\mathrm{F}_{\mathrm{in}} - \mathrm{F}_{\mathrm{ex}})\frac{\rho_1(\boldsymbol{r}_1) + \rho_2(\boldsymbol{r}_2)}{\rho_2(0) + \rho_2(0)}\right]\delta(\boldsymbol{r}_{12}), \qquad F_{\mathrm{ex(in)}} = f_{\mathrm{ex(in)}} \pm f'_{\mathrm{ex(in)}}$$

C, MeV·fm^{-3}	f_{in}	f_{ex}	f_{in}'	f_{ex}'
300	0.09	-2.59	0.42	0.54

A. B. Migdal. The Theory of Finite Fermi-Systems and Properties of Atomic Nuclei, 2-nd ed. (Nauka, Moscow, 1983) [in Russian].

Bass potential:

$$V_{\mathrm{Bass}}(R) = \frac{Z_1 Z_2 e^2}{R} - \frac{R_1 R_2}{R_1 + R_2} g(\xi) \qquad g(\xi) = \left[A\exp\left(\frac{\xi}{d_1}\right) + B\exp\left(\frac{\xi}{d_2}\right)\right]^{-1}$$

$$\xi = R - (R_1 + R_2)$$

A, MeV^{-1}·fm	B, MeV^{-1}·fm	d_1, fm	d_2, fm
0.03	0.0061	3.3	0.65

R. Bass. Nuclear Reactions with Heavy Ions. Berlin: Springer, 1980.

Density parameters

					a_p (+Pb-208)		R0_p		R0_n		R_p_rms		R_n_rms		R_ch_rms
Z	A	rho0_p	rho0_n	rho_tot	Old	New	Old	New	Old	New	Old	New	Old	New	
8	16	0.1002	0.1061	0.2063	0.52	0.558	2.340	2.290	2.283	2.232	2.650	2.729	2.620	2.700	2.759
20	40	0.0869	0.0900	0.1769	0.52	0.590	3.568	3.502	3.521	3.454	3.373	3.488	3.343	3.459	3.459
20	48	0.0777	0.0955	0.1732	0.52	0.588	3.720	3.658	3.905	3.846	3.470	3.578	3.590	3.695	3.554
28	60	0.0793	0.0880	0.1674	0.52	0.607	4.181	4.108	4.227	4.154	3.772	3.900	3.802	3.929	3.849
40	90	0.0734	0.0867	0.1601	0.52	0.628	4.891	4.811	4.992	4.914	4.253	4.397	4.323	4.465	4.322
50	124	0.0672	0.0896	0.1568	0.52	0.639	5.463	5.382	5.667	5.589	4.652	4.799	4.797	4.939	4.716
62	144	0.0686	0.0851	0.1537	0.52	0.658	5.849	5.760	5.982	5.895	4.926	5.088	5.021	5.179	4.985
82	208	0.0632	0.0883	0.1515	0.52	0.673	6.635	6.547	6.856	6.770	5.491	5.654	5.651	5.809	5.544
36	86	0.0716	0.0902	0.1617	0.52	0.608	4.753	4.687	4.922	4.858	4.158	4.277	4.275	4.390	4.229
54	136	0.0660	0.0899	0.1559	0.52		5.650		5.868		4.784		4.940		4.846

Radius parametrization

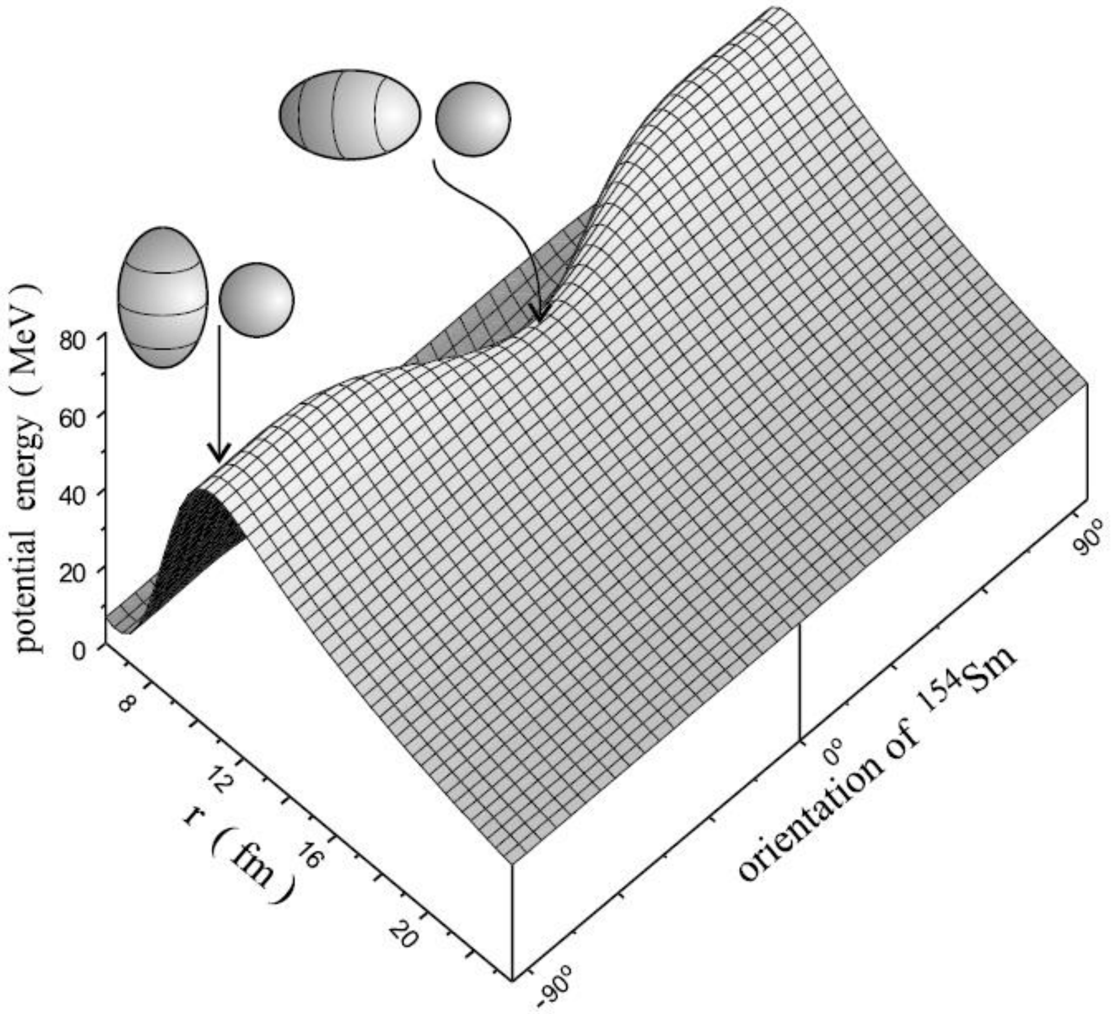
New parametrization of available in literature analytical formulas for rms charge radius $\langle r^2 \rangle_{ch}^{1/2}$ for nuclei with $Z \geq 8$ and $A \geq 16$, based on experimental data

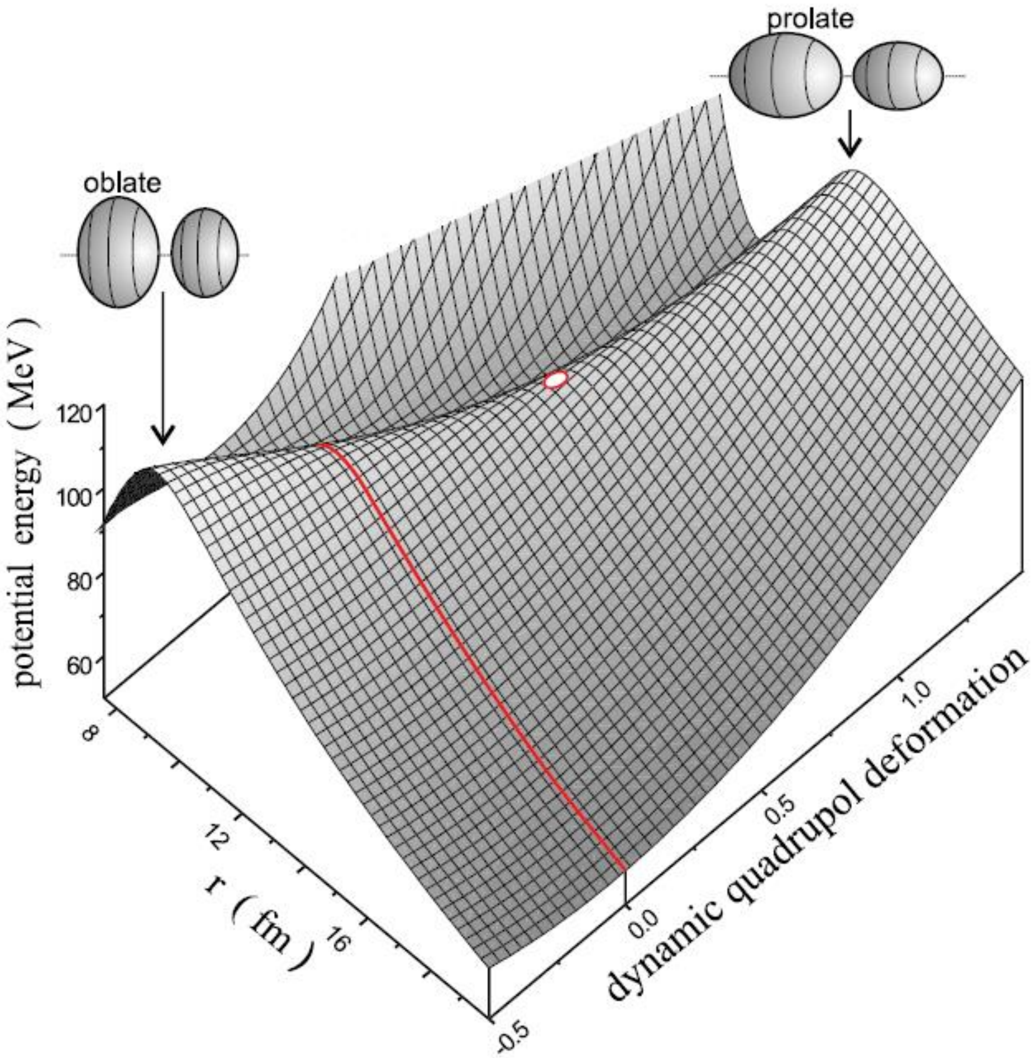
	Формула	Параметры	Станд. откл. σ, фм
	$R = r_A A^{1/3}$	$r_A = 0.9524(7)$	0.095
	$R = r_f Z^f$	$r_f = 1.357(7),\ f = 0.3168(12)$	0.067
	$R = r_f A^f$	$r_f = 1.1446(56),\ f = 0.2957(10)$	0.059
Z	$R_{rms} = (a + \frac{b}{c^2+Z^2})A^{1/3}$	$a = 0.938(10),\ b = 36(3),\ c^2 = 223(5)$	0.059
	$R_s = \left(r_{0,s} + \frac{r_{1,s}}{A^{2/3}} + \frac{r_{2,s}}{A^{4/3}}\right) \cdot A^{1/3}$	$r_{0,s} = 0.9207(25),\ r_{1,s} = 0.613(9),\ r_{2,s} = 3.59(67)$	0.051
D	$R_{\alpha,\alpha^2} = a + bA^{1/3} + c\alpha + dA^{1/3}\alpha^2$	$a = 0.484(10),\ b = 0.880(2),\ c = -1.098(56),\ d = 0.338(42)$	0.041
NP	$R_{\alpha,A} = r_A \left(1 - b\alpha + c\frac{1}{A}\right) A^{1/3}$	$r_A = 0.9560(14)$ фм, $b = 0.1527(67),\ c = 2.326(63)$	0.041

Neutron excess $\alpha = (N - Z)/A$

Exp. data: I. Angeli et al. At. Data Nucl. Data Tables 99, 69 (2013).
Fit D: J. Duflo. Nuclear Physics A 576 (1994).
Fit NP: B. Nerlo-Pomorska et al. Zeitschrift f¨ur Physik A 348 (1994).

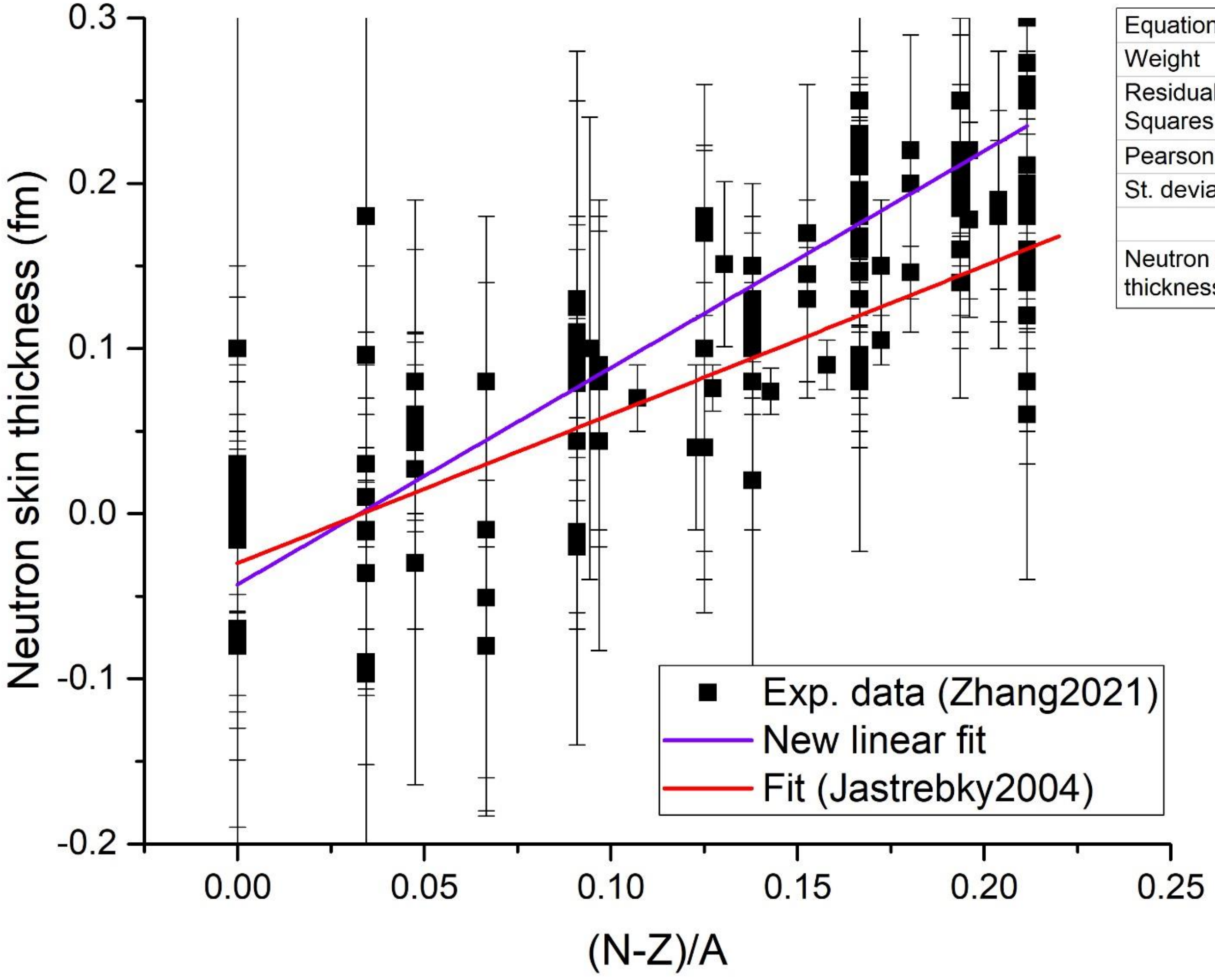
Impact of deformation





Source: V. Zagrebaev, W. Greiner. Nucl. Phys A 944(2015)257–307

New data on the neutron skin



Equation	y = a + b*x		
Weight	Direct Weighting		
Residual Sum of Squares	0.05699		
Pearson's r	0.81311		
St. deviation	0.02078		
		Value	Standard Error
Neutron skin thickness	Intercept	-0.04298	0.01077
	Slope	1.31238	0.08178

***Old fit (red)**: J. Jastrzebski et al. IJMP E, v. 13. (2004)*
New data and fit: J. Zhang et al. Phys. Rev. C 104, 034303 (2021)