SOLITON SOLUTIONS OF HYDRODYNAMIC EQUATIONS IN DESCRIBING COLLISIONS AND OSCILLATIONS OF ATOMIC NUCLEI

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Within the framework of the non-equilibrium hydrodynamic approach, a soliton-like analytical solution of the equations for the collision of nuclear layers-slabs is found. The prospects of the hydrodynamic approach to the description of collisions of heavy ions of medium energies and the importance of taking into account non-equilibrium processes are noted. Within the framework of a single formula, the stages of compression, expansion and expansion of layers of nuclear matter with energies of the order of ten MeV per nucleon are considered. Here we develop an approach to the approximate analytical solution of these equations, both in the case of weak nonlinearity, by reducing them to the Korteweg-de Vries equations, and in the case of large-amplitude perturbations, using soliton-like solutions. Our generalization to the two-dimensional case leads to the idea of the formation of a rarefied bubble region at the stage of expansion.

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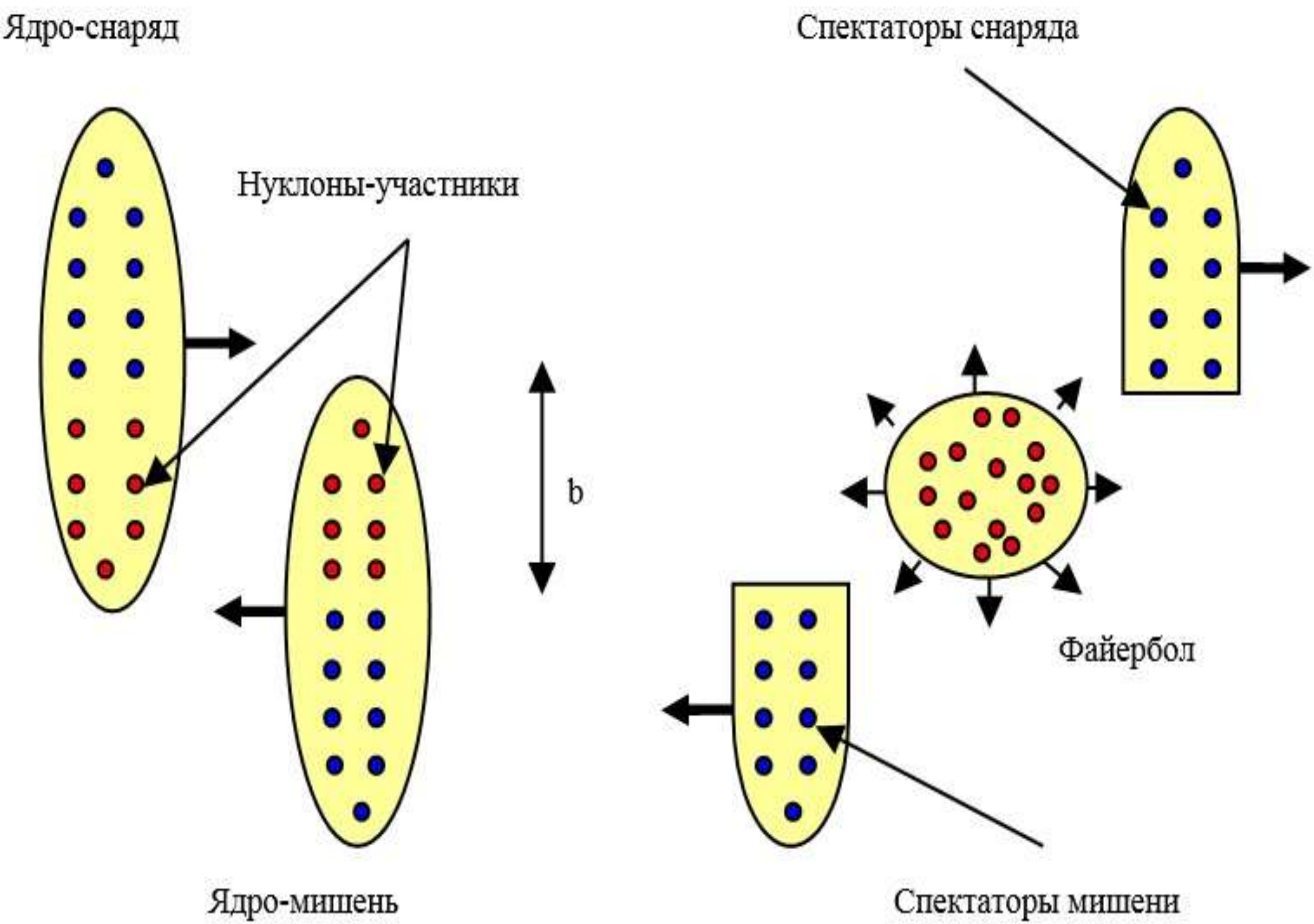
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1. FEATURES OF NON-EQUALIBRIUM HYDRODYNAMIC APPROACH

- To describe the collisions of heavy ions we use the non-equilibrium hydrodynamic approach, in which the kinetic equation for the nucleon distribution function $f(\mathbf{r},\mathbf{p},t)$ is solved jointly with the equations of hydrodynamics, which are essentially local laws of conservation of mass, momentum and energy. $\int \frac{d^3\vec{p}}{(2\pi\hbar)^3} \quad 1, \vec{p}, p^2$
-
- $$\frac{df}{dt} = \frac{f_0 - f}{\tau} \quad , \qquad (1)$$
-
-
- where $f_0(\mathbf{r},\mathbf{p},t)$ is the locally equilibrium distribution function and
- ‘ τ is the relaxation time
- $$W(\rho) = \alpha\rho + \beta\rho^{\chi} \quad , \qquad (2)$$
-

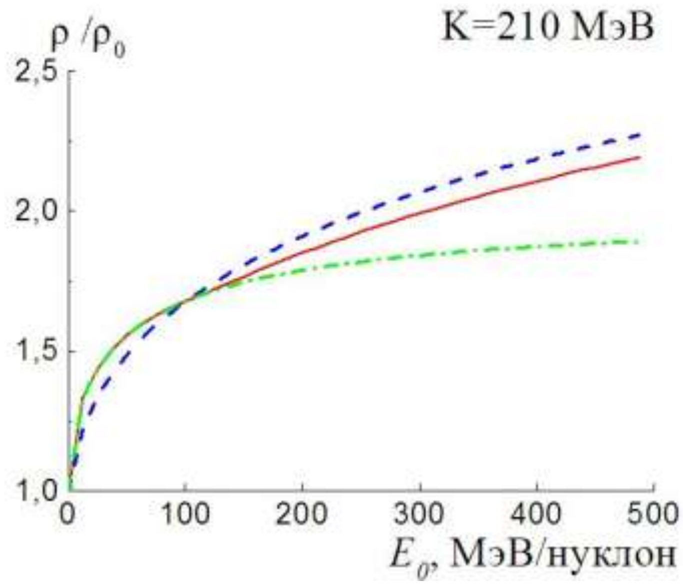
- The self-consistent potential appearing in the interaction term is specified in just the same way as this is done in the case of density-dependentive forces belonging to the Skyrme type

- $\rho = \int \frac{f d^3\vec{p}}{(2\pi\hbar)^3}$ $\qquad \tau = \lambda / \upsilon_T \qquad W(\rho) = \alpha\rho + \beta\rho^{\gamma},$
- $$f(\vec{r}, \vec{p}, t) = f_1 \cdot q + f_0 \cdot (1-q) \qquad (3)$$
-
-
- $$P_{kin}^{\parallel} = P_{(kin)11} = 2(\varepsilon_1 + I_1)q + \frac{2}{3}(\varepsilon + I)(1-q) \qquad \varepsilon_1 = \frac{\hbar^2}{10m}\left(\frac{3}{2}\pi^2\rho_0\right)^{2/3}\frac{\rho^3}{\rho_0^2}$$
- $$P_{kin}^{\perp} = P_{(kin)22} = P_{(kin)33} = 2\varepsilon_2 + \frac{2}{3}(\varepsilon + I)(1-q) \qquad \varepsilon_2 = \frac{\hbar^2}{10m}\left(\frac{3}{2}\pi^2\rho_0\right)^{2/3}\rho$$
- $$\varepsilon = \frac{3}{10}\frac{\hbar^2}{m}\left(\frac{3}{2}\pi^2\rho\right)^{2/3}\cdot\rho \qquad I = \int \frac{p^2}{2m}\delta f \frac{d^3\vec{p}}{(2\pi\hbar)^3} \qquad I_1 = \int \frac{p^2}{2m}\delta f_1 \frac{d^3\vec{p}}{(2\pi\hbar)^3}$$
- $$P_{ij} = P_{(kin)ij} + P_{int}\delta_{ij} \qquad e = \varepsilon + I + e_{int} \quad e_{int} = \int_0^{\rho} W(\rho)d\rho \qquad P_{int} = \rho^2 \frac{d(e_{int}/\rho)}{d\rho}$$
- $$\frac{\partial}{\partial t}((\varepsilon_1 - \varepsilon_2 + I_1)q) + \frac{\partial}{\partial x_1}(\upsilon_1(3\varepsilon_1 - \varepsilon_2 + 3I_1)q) + \sum_{i=2,3}\frac{\partial}{\partial x_i}(\upsilon_i(\varepsilon_1 - \varepsilon_2 + I_1)q) +$$
- $$\rho\upsilon_1\frac{\partial W}{\partial x_1} - \sum_{i=2,3}\frac{\rho\upsilon_i}{2}\frac{\partial W}{\partial x_i} = -\frac{(\varepsilon_1 - \varepsilon_2 + I_1)q}{\tau} \qquad (4)$$

$p_1^2 - (p_2^2 + p_3^2)/2$

2.THE HYDRODYNAMIC STAGE

- After selecting the region of the local heating, hot spot - the overlap region of the colliding nuclei, we analyze the stages of compression, expansion and freeze-out of matter during the collision of heavy ions. At the compression stage, a collisionless shock wave with a changing front is formed. At the expansion stage, when the shock wave reaches the boundaries of hot spot, the initially compressed system is expanded, we describe it in the relaxation approximation taking into account the nuclear viscosity. As the relaxation time we take $\tau = \lambda / \upsilon_T$ where $\lambda = 1/\sigma\rho$ is the mean free path, $\sigma \approx$ 40mb is the total nucleon-nucleon cross section, ρ is the nucleon density, and
- υ_T is the average velocity of the thermal motion of the nucleons. At the freeze-out stage, when the system reaches a critical density also called the freeze-out density, the system does not "hold itself" and the secondary particles are formed.



- Fig. 1. Dependence on the collision energy of the maximum compression ratio
- ρ/ρ_0 achieved in the central collision of nuclei for the case of the relaxation factor
- q (solid line) calculated by us, for the case when the factor $q = 0$
- (dashed line), and for the case when $q = 1$ (a dashed-dot line)

3.KORTEVEG-DE VRIES SOLITONS

$$\frac{\partial \rho}{\partial t} + \upsilon \frac{\partial \rho}{\partial x} + \rho \frac{\partial \upsilon}{\partial x} = 0 \qquad P(\rho) = a\rho + b\rho^2 + \acute{\alpha}\frac{d^2\rho}{dx^2} \qquad a\rho_0 + b\rho_0^2 = 0 \qquad I = I_1\left(\frac{\rho}{\rho_0}\right)^3$$

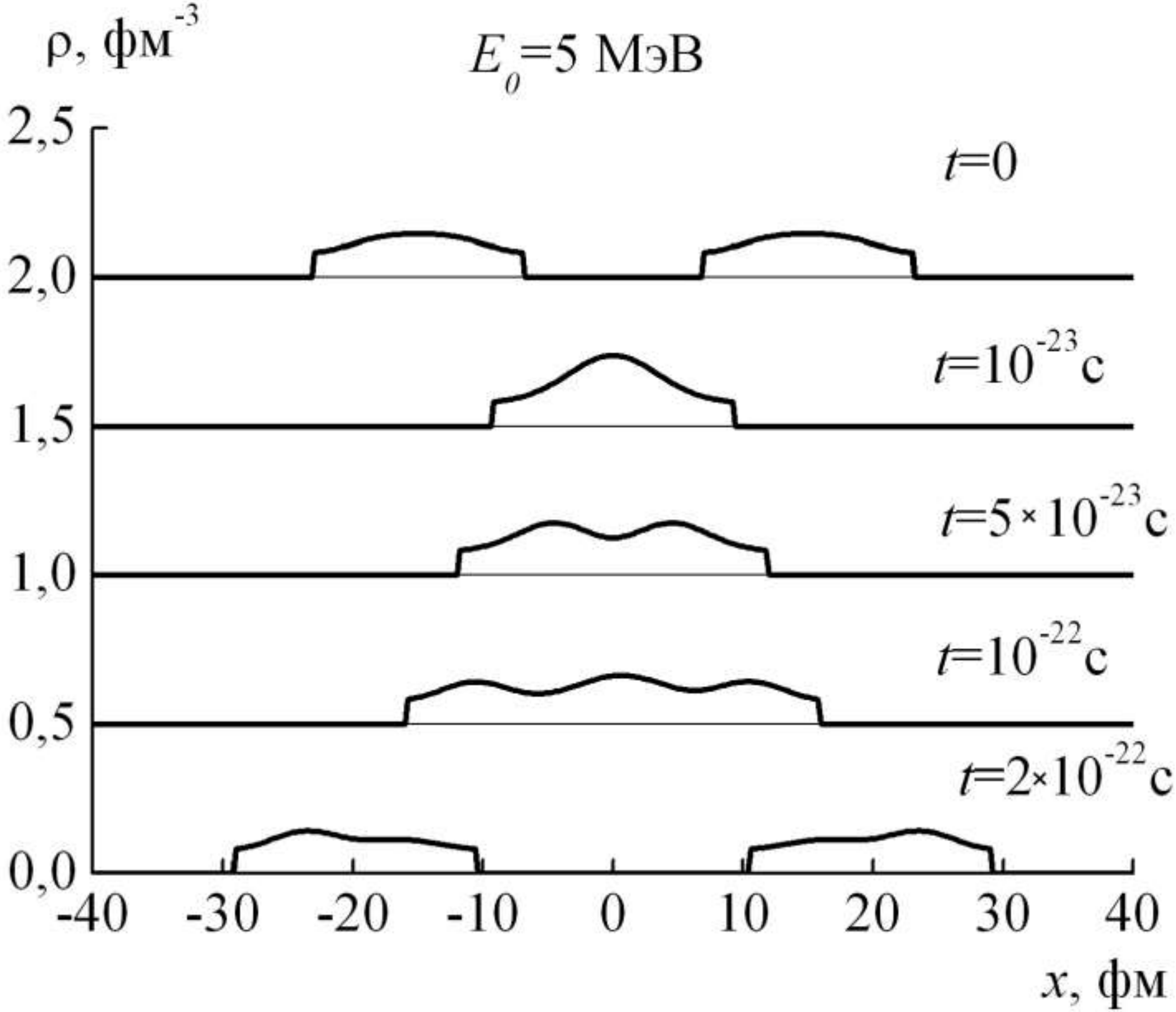
$$\frac{\partial \upsilon}{\partial t} + \upsilon \frac{\partial \upsilon}{\partial x} + \frac{1}{m\rho}\frac{\partial P}{\partial x} = 0 \qquad \rho\upsilon'(\rho) = \pm\sqrt{\frac{\partial P}{m\partial \rho}} \approx \pm\left[c_{so} + \beta(\rho - \rho_0) + \frac{\alpha}{2mc_{so}}\frac{\partial}{\partial \rho}\left(\frac{\partial^2 \rho}{\partial x^2}\right)\right] = \pm c_s(\rho)$$

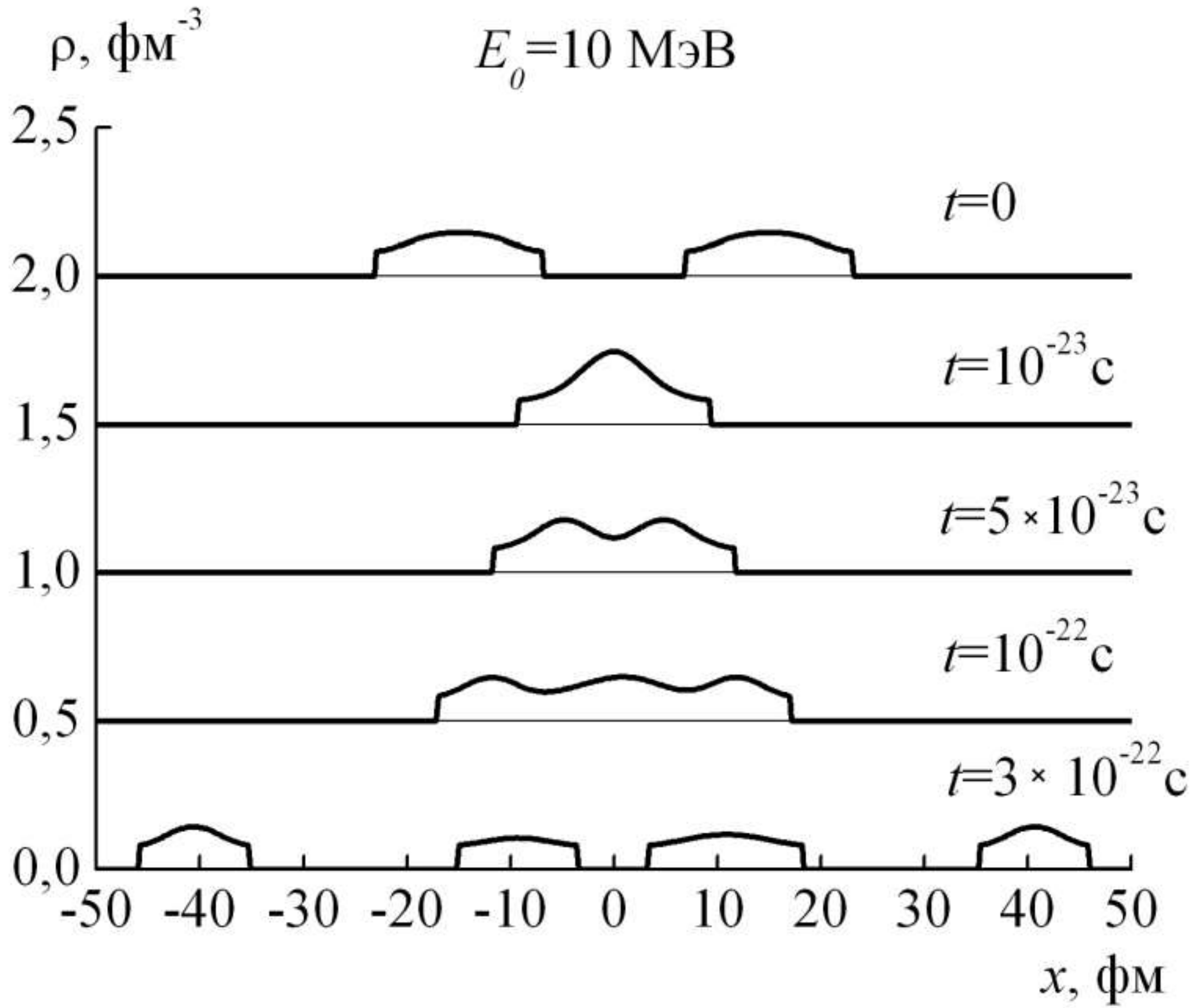
$$\frac{\partial I}{\partial t} + \upsilon \frac{\partial I}{\partial x} + 3I\frac{\partial \upsilon}{\partial x} = 0 \qquad \upsilon = \pm\int_{\rho_0}^{\rho}\frac{c_s(\rho)}{\rho}d\rho + \upsilon_0 \qquad c_{s0} = \sqrt{\frac{a + 2b\rho_0}{m}} \approx 1/3c$$

$$\frac{\partial \zeta}{\partial t'} + 6\zeta\frac{\partial \zeta}{\partial x'} + \frac{\partial^3 \zeta}{\partial x'^3} = 0 \qquad \zeta = \left[\pm\upsilon_0 + c_{so} + 2c_{so}\frac{(\rho - \rho_0)}{\rho_0}\right]\frac{1}{A} \qquad \zeta = 2\frac{\partial^2 \ln s}{\partial x'^2}$$

$$s = 1 + \exp(\omega t' + k(x' - x_1)) \qquad Z = \int_0^L \xi \frac{dx_1}{L}$$

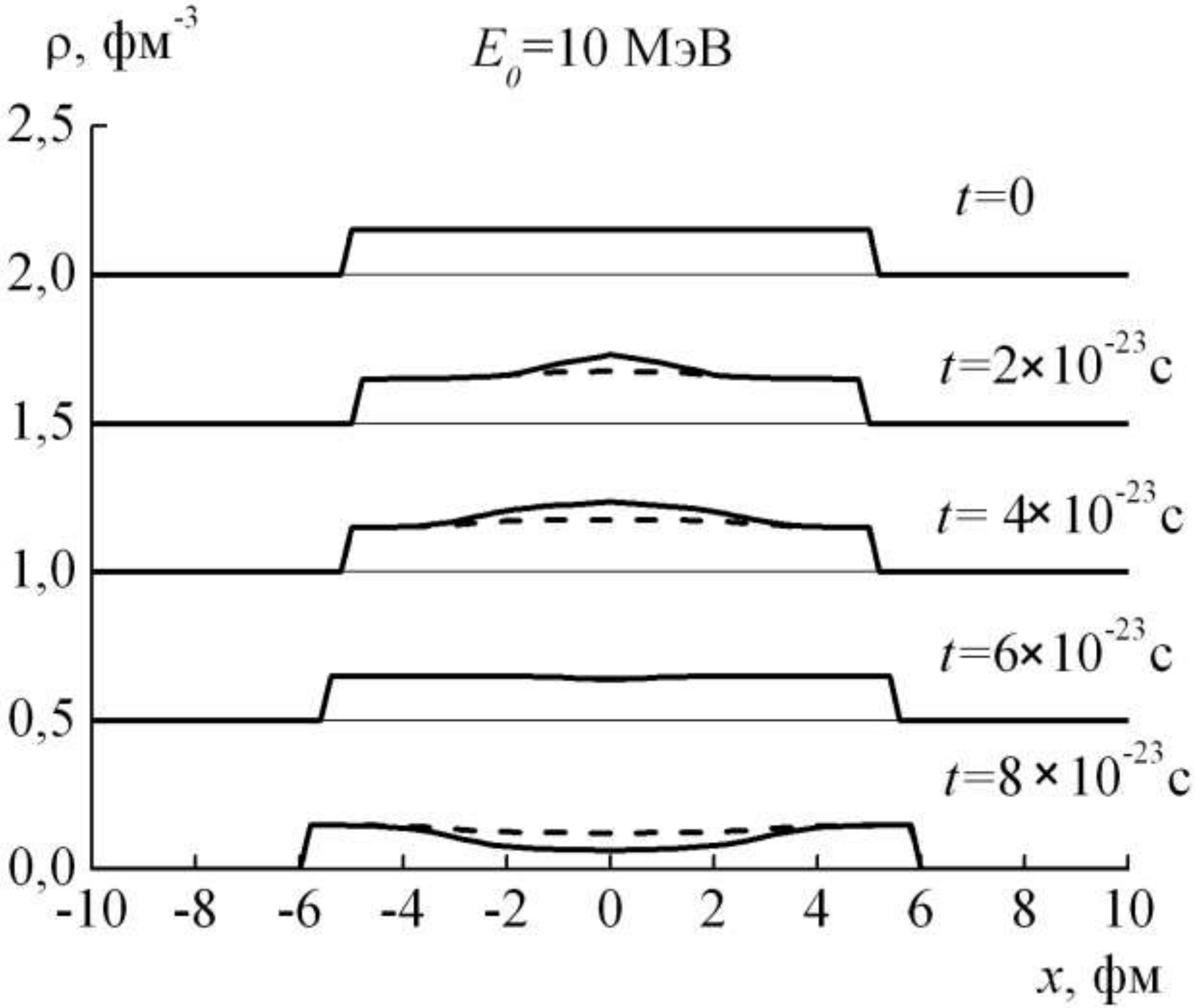
$$\omega = -k^3$$


ρ, фм$^{-3}$
E_0=5 МэВ
2,5
2,0
1,5
1,0
0,5
0,0
t=0
t=10^{-23}с
t=5×10^{-23}с
t=10^{-22}с
t=2×10^{-22}с
-40
-30
-20
-10
0
10
20
30
40
x, фм

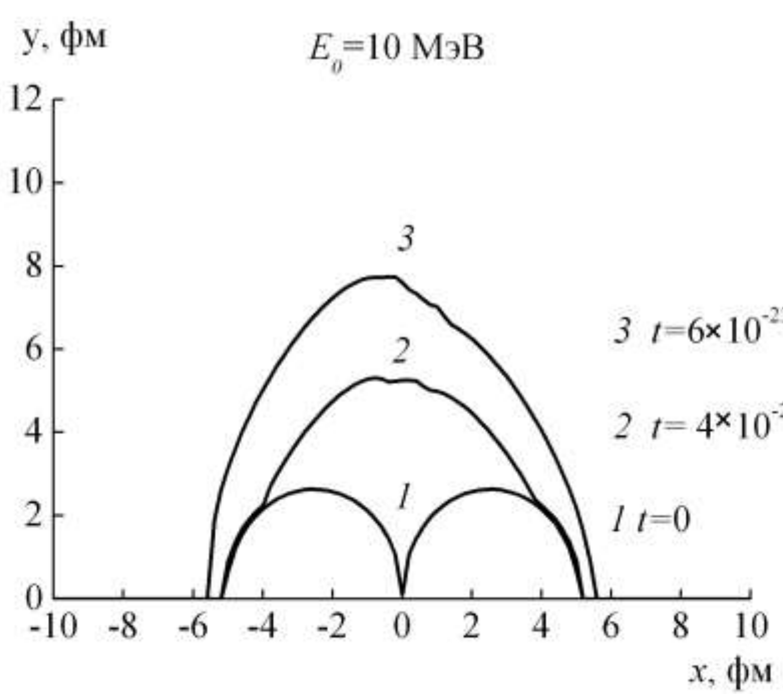


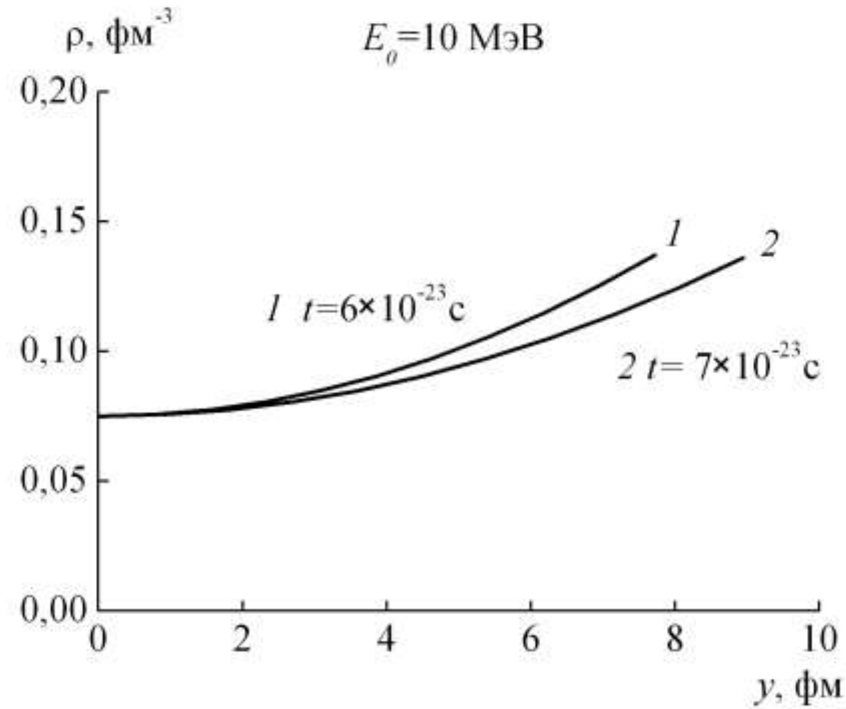
ρ, фм^-3
E0=10 МэВ
2,5
2,0
1,5
1,0
0,5
0,0
t=0
t=10^-23 c
t=5 × 10^-23 c
t=10^-22 c
t=3 × 10^-22 c
-50
-40
-30
-20
-10
0
10
20
30
40
50
x, фм

Shock wave formation



Formation of bubble



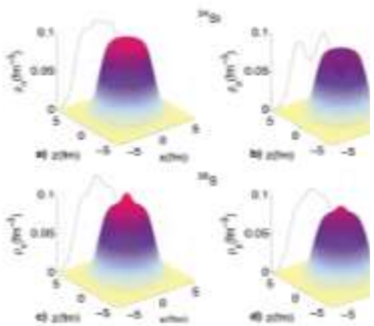


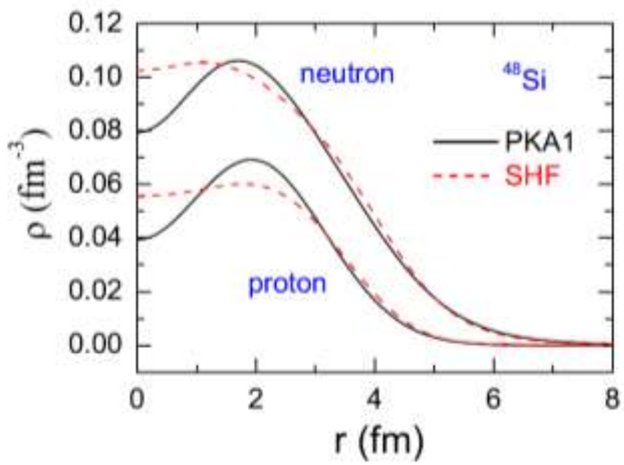
$$D = -\frac{\rho_0 \upsilon_0}{\rho - \rho_0} \qquad P = -\frac{\partial(e/\rho)}{\partial(1/\rho)} = K(\rho^2 - \rho_0^2) - \alpha\left(\frac{\partial\rho}{\partial x}\right)^2 \qquad c_s = \sqrt{\frac{\partial P}{m\partial\rho}} = D$$

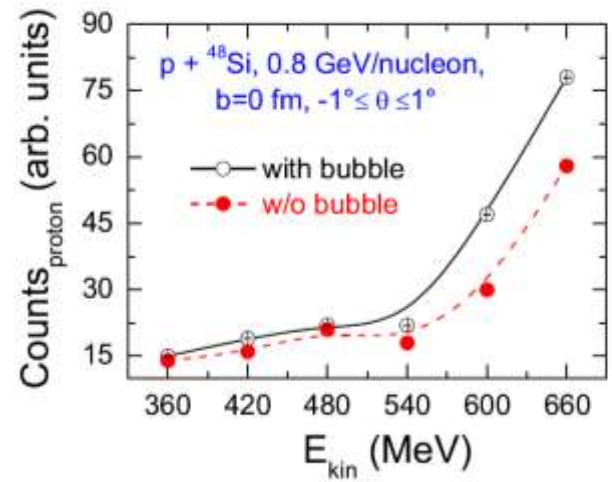
$$\rho = \rho_0 + 4\frac{(\rho_1 - \rho_0)}{(\exp(-\lambda x/2) + \exp(\lambda x/2))^2} \qquad 2K\rho_1 = \frac{(\rho_0\upsilon_0)^2}{(\rho_1 - \rho_0)^2}$$

J. Decharge, J.-F. Beger, K. Dietrich, and M.S. Weiss, Phys. Lett. B 451 275 (1999). **Z>120**

Probing bubble configuration







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Nature Physics, ***v 13.*** *p 152–156 (2017)*

X.H Fam. G.C. Yong, W. Zuo.
Probing nuclear bubble configuration by proton induced reaction
Phys.Rev.C 99 (2019) 4, 041601

4. A COMPARISON WITH EXPERIMENTAL DATA

- As a result, the double differential cross-section of proton emission has the form (where *b* is the impact parameter, $\hbar$ is a Planck constant, $\vec{r}$ is the radius vector):
- $$E\frac{d^2\sigma}{p^2dpd\Omega}=\frac{2\pi}{(2\pi\hbar)^3}\int G(b)bdbd\vec{r}\gamma(E-\vec{p}\vec{\upsilon})f(\vec{r},\vec{p},t) \quad (5)$$
- where the distribution function of emitted protons
- $$f(\vec{r},\vec{p},t)=g\left[\exp\left(\frac{\gamma(E-\vec{p}\vec{\upsilon}-\mu)+T\delta}{T}\right)+1\right]^{-1} \quad (6)$$
- Here the spin factor $g=2$, $E=\sqrt{p^2+m^2}$, γ and $\vec{p}$ are respectively the total energy, the Lorentz factor and the proton momentum; $\vec{\upsilon}(\vec{r},t)$ is the velocity field, $G(b)$ is the factor taking into account that the cross section of the hot spot formation is always greater than the geometric one, μ is the chemical potential, which is found from the conservation of the average number of particles for a grand canonical ensemble, T is the temperature, δ is correction for the microcanonical distribution.

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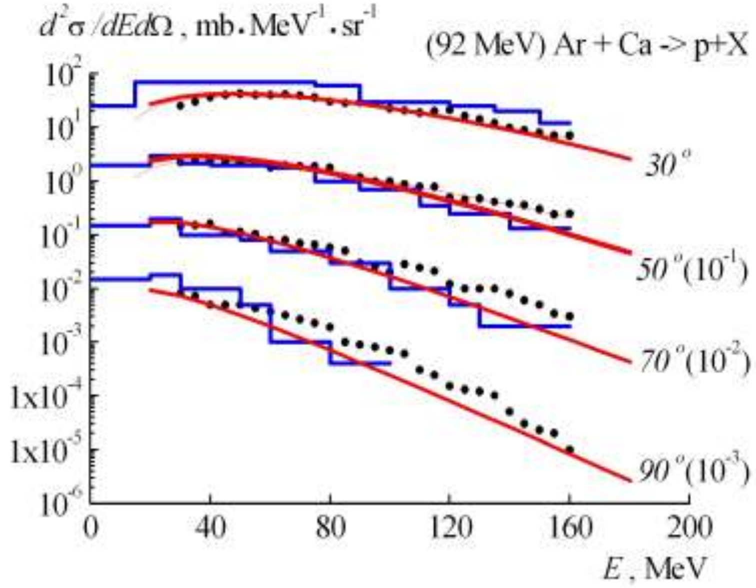
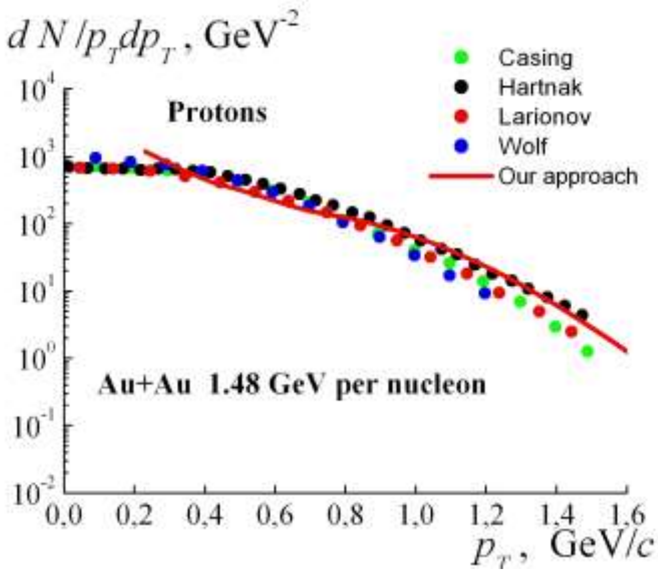
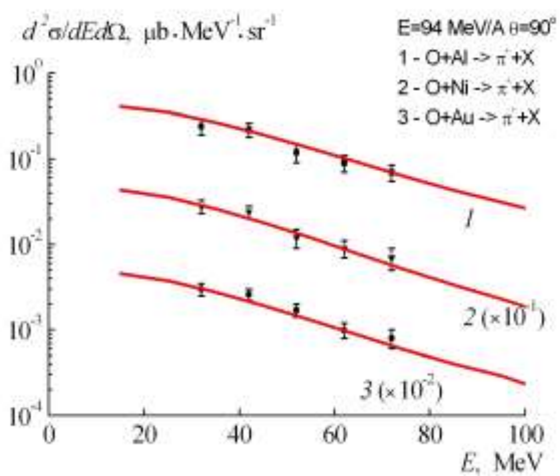
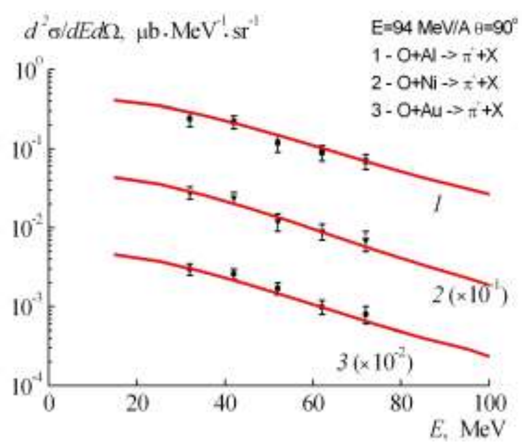
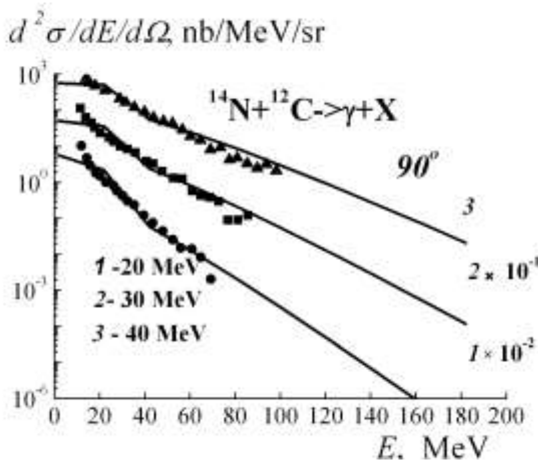
Protons, pions and photons

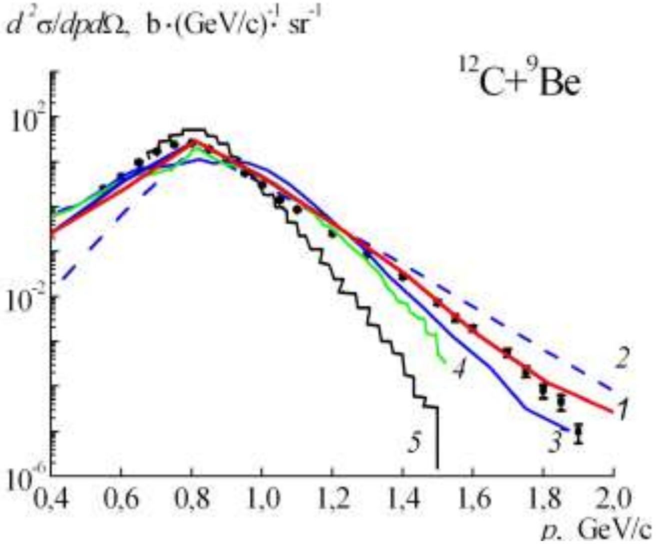




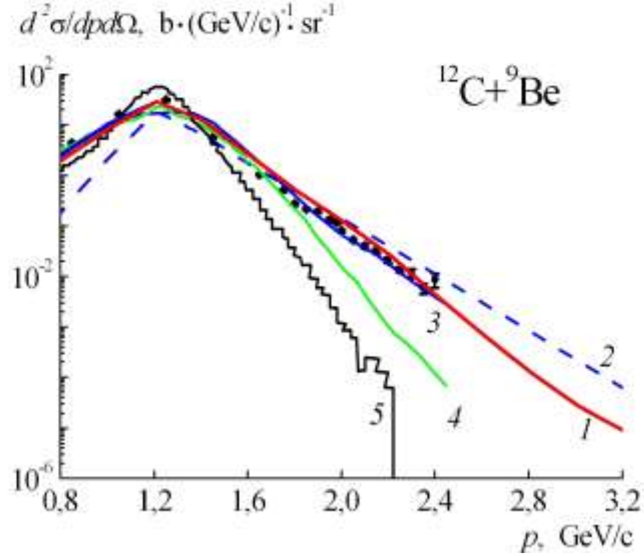




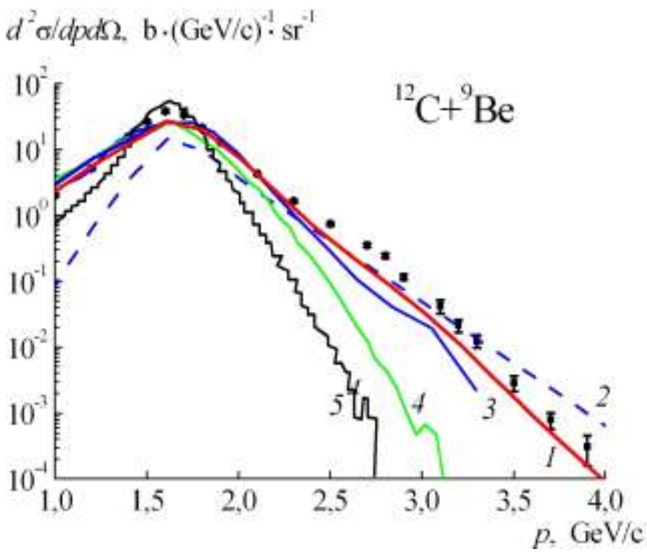




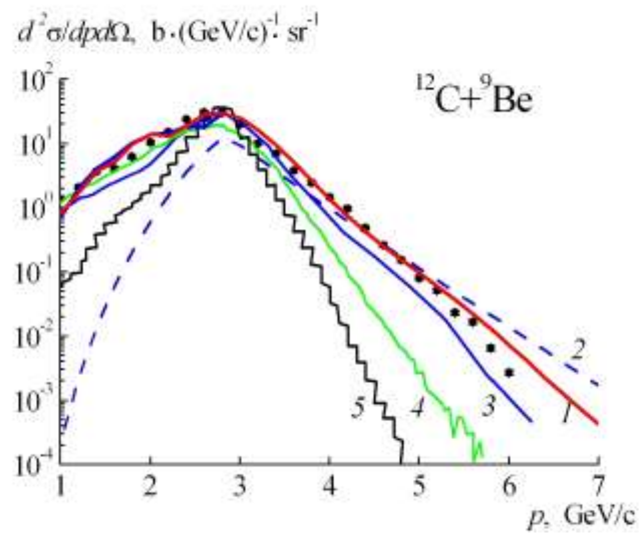
$^{12}C+^{9}Be$ 300 MeV/nucl.
(protons) $3{,}5^{0}$



$^{12}C+^{9}Be$ 600 MeV/nucl.
(protons) $3{,}5^{0}$



$^{12}C+^{9}Be$ 950 MeV/nucl.
(protons) $3{,}5^{0}$



$^{12}C+^{9}Be$ 2000 MeV/nucl.
(protons) $3{,}5^{0}$

Conclusion

- Thus, in this paper, the idea of using of a non-equilibrium equation of state in the hydrodynamic approach to describe the high-momentum proton, pion and photon spectra emitted in heavy-ion collisions over a wide energy range has been further developed. We also succeded in the description of the subthreshold pion energy spectra.
- The introduction of dispersion into the effective forces and into the pressure does not violate the concept of the formation of a hot spot and a bubble configuration. The introduction of additional dimensions does not violate this representation. And the approach itself is of independent interest and can be used in other areas of physics when calculating the nonlinear dynamics of oscillations of complex systems.

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THANK YOU FOR ATTENTION!