PECULIARITIES OF THE ENERGY SPECTRUM OF THE ^{12}C NUCLEUS IN A 3α MODEL

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E. M. Tursunov, J. Phys. G: Nucl. Part. Phys. **27**, 1381 (2001).
E. M. Tursunov, Pramana–J. Phys. **95**, 128 (2021).
E.M. Tursunov, I. Mazumdar Phys. Atom Nucl. 85 (2022), pp.160-166.

Nucleus -2022

CONTENTS

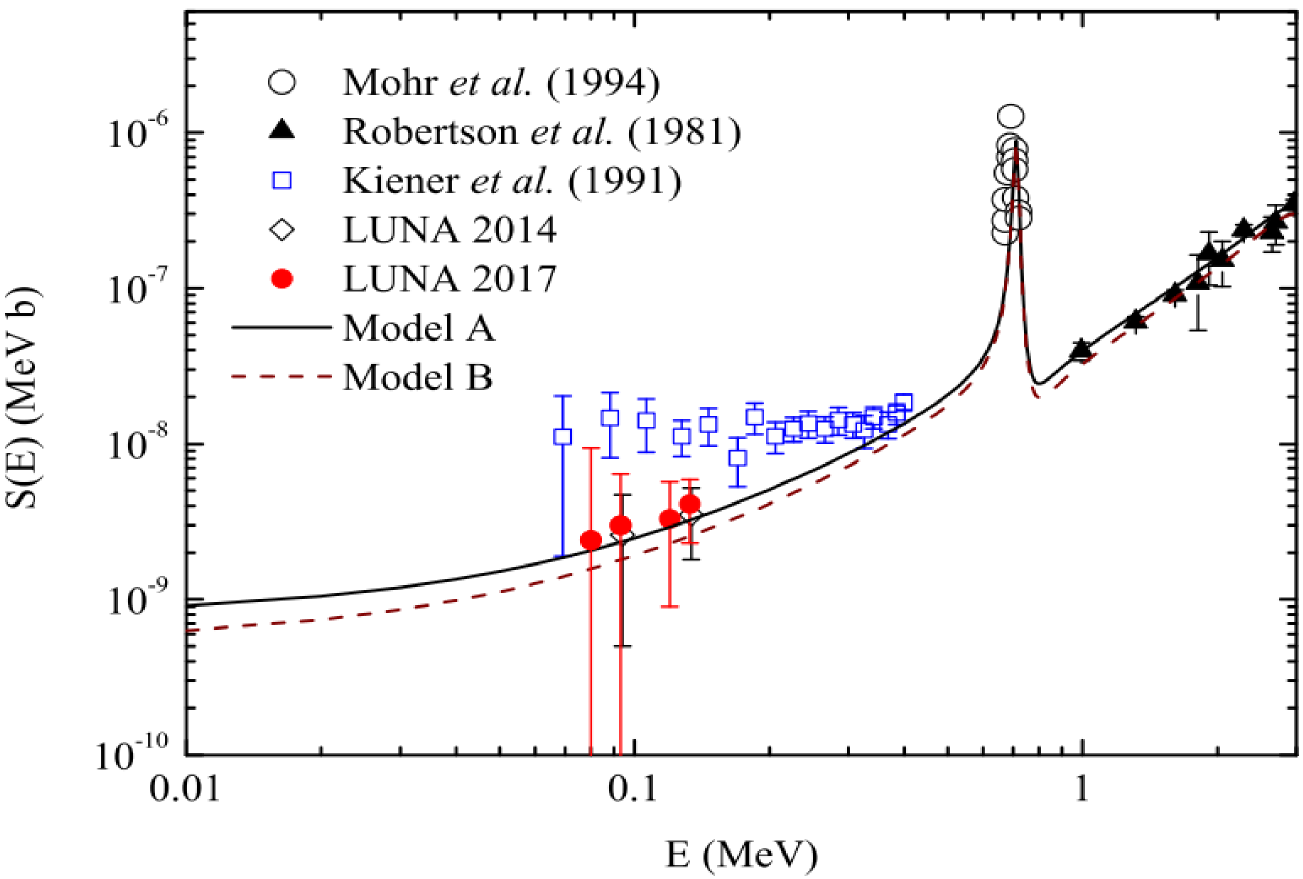
Motivation

Three-body models for systems with a single α particle

^{6}**Li=** $\alpha+n+p$, ^{6}**He=** $\alpha+n+n$

work very well. ***αN*** *and* ***NN*** *potentials describing 2-body data (phase shifts and binding energy of the deutron) with small additional 3-body potentials yield energies of the three-body systems and reaction cross sections.*

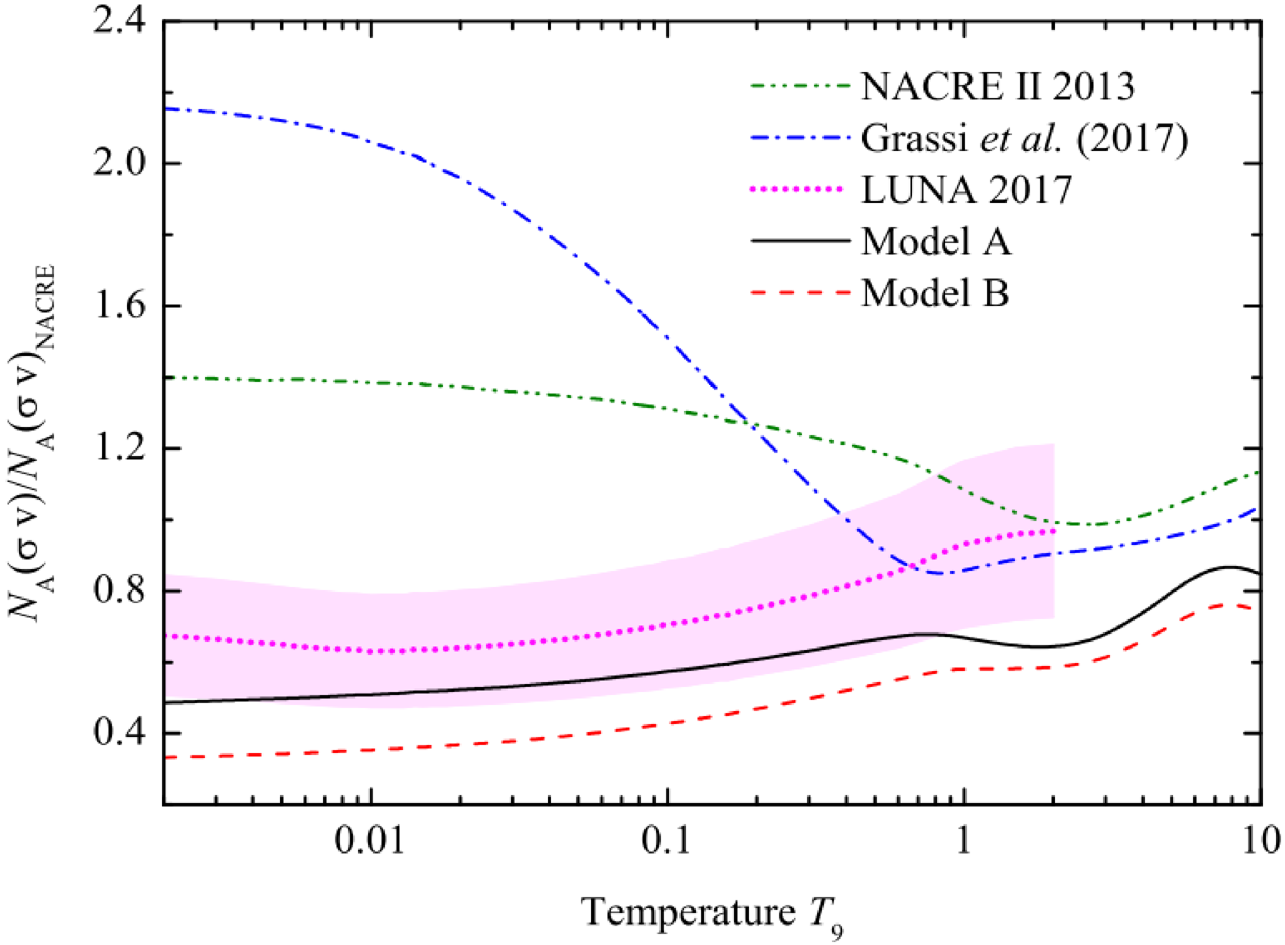
Astrophysical S-factor for the $\alpha(d,\gamma)^6Li$ capture process



D. Baye, E. Tursunov, ***J. Phys. G: Nucl. Part. Phys***. 2018, 45(8), 085102;

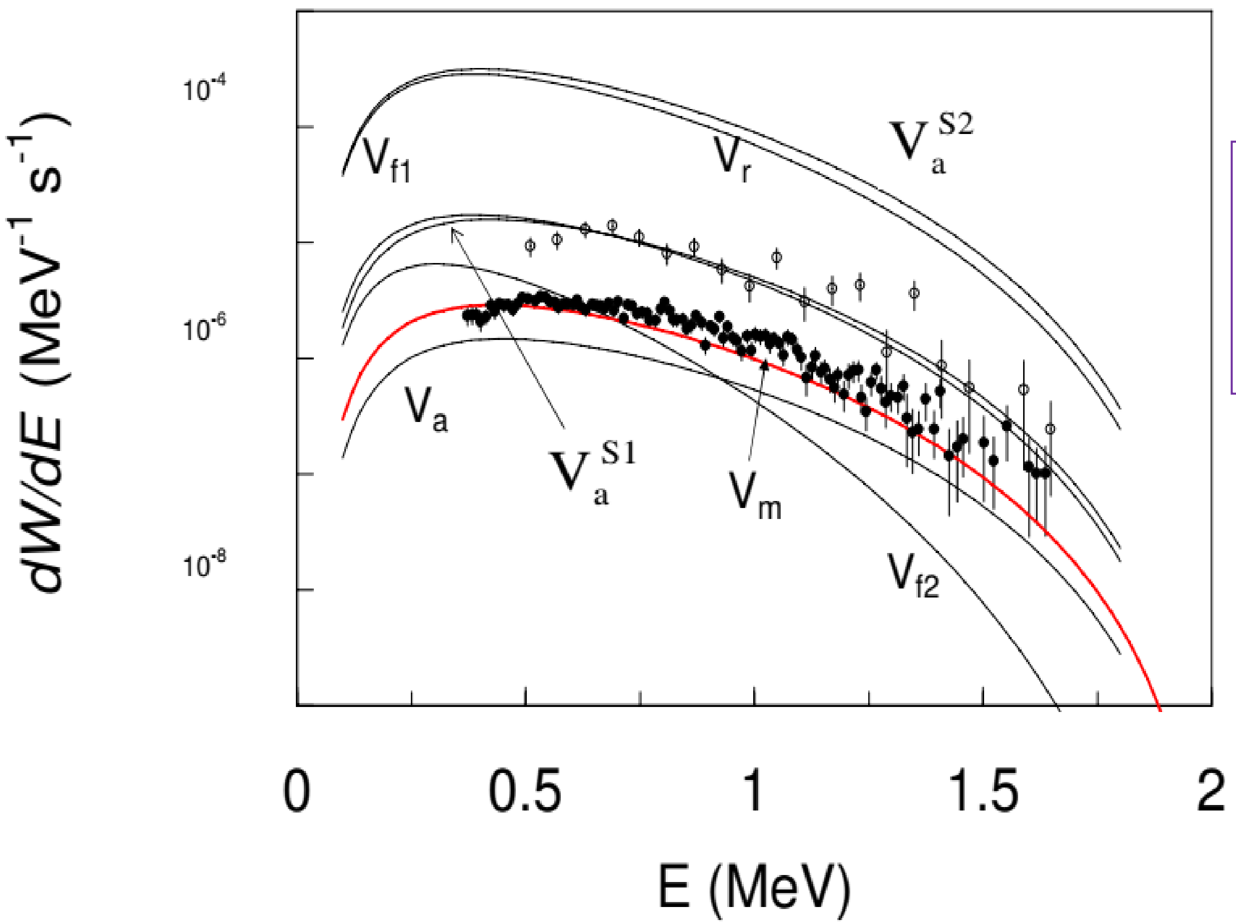
E. Tursunov ***et al.*** ***Phys. Rev.C***, 2018, 98(5), 055803

Reaction rates of the $\alpha(d,\gamma)^6Li$ astrophysical synthesis process



E. Tursunov *et al.* ***Physical Review C***, 2018, 98(5), 055803

Transition probability per time and energy units *dW/dE* of the ^{6}He *beta* decay into the $\alpha + d$ continuum for different $\alpha + d$ potentials



E.M. Tursunov, D. Baye, P. Descouvemont, ***Phys. Rev. C*** , 2006, 73(1), 014303

But, there is a big problem in the $^{12}C=3\alpha$ model
a) Potential ABd0 with a repulsive core

S. Ali, A.R. Bodmer,
Nucl. Phys. A80
(1966) 99

$$V_{AB}(r) = V_1 exp(-\eta_1 r^2) + V_2 exp(-\eta_2 r^2) + V_{coul}(r),$$

$$V_1 = 500 \text{ MeV}, \ \eta_1 = 0.49 \ fm^{-2} \qquad V_{Coul}(r) = 4e^2 erf(br)/r$$

$$V_2 = -130 \text{ MeV}, \ \eta_2 = 0.225625 \ fm^{-2} \qquad \text{b=0.75 fm}^{-1}$$

b) l-dependent potential ABd with a repulsive core

$$V_1 = 320 \text{ MeV for } l = 2, \ V_1 = 0 \text{ for } l > 2 \qquad b = \sqrt{3}/2.88 \text{ fm}^{-1}$$

7

Energies of the ^{12}C(0+) states in a 3α model with potentials with a repulsive core

(J^π, T)	LMM	ABd$_0$ (l-indep.) HHM	VGM	
$(0_1^+, 0)$	-0.58427008	-0.58407	-0.584266	with Coulomb
$(0_1^+, 0)$	-5.122093595	-5.122	-5.1220936	no Coulomb
$(0_2^+, 0)$	-1.3606	-1.2	-1.36062	
$(0_3^+, 0)$	-1.338	-0.8	-1.33873	
$(2_1^+, 0)$		-1.15	-1.3398	
		ABd (l-dep.)		
$(0_1^+, 0)$		-1.523	-1.523	with Coulomb
$(0_1^+, 0)$		-6.423	-6.423	no Coulomb
$(0_2^+, 0)$		-1.92	-1.934	

Exp.

$(0_1^+, 0)$	-7.275
$(0_2^+, 0)$	0.3796
$(2_1^+, 0)$	-2.836

E. Tursunov,
D. Baye,
P. Descouvemont,
Nucl. Phys. A723
(2003), 365–374

8

$\alpha\alpha$- potential of BFW with forbidden states in S and D waves

$$V(r) = V_0 \exp\left(-\eta r^2\right) + 4e^2 erf(br)/r,$$

B. Buck, H. Friedrich, C. Wheatley. Nucl. Phys. A275 (1977) 246

$$V_0 = -122.6225\ \mathrm{MeV}, \eta = 0.22\ \mathrm{fm}^{-2}$$

$$b = 0.75\ \mathrm{fm}^{-1}$$

Very good description of the experimental phase shifts $\delta_L(E)$ for the $\alpha\alpha$-elastic scattering in the partial waves L = 0, 2, 4 within the energy range up to E=40 MeV and the energy positions and widths of the ^8Be resonances.

Pauli forbidden states in the S wave with energies E1 = −72.6257 MeV and E2 = −25.6186 MeV, and a single forbidden state in the D wave with E3 =−22.0005 MeV.

A) Method orthogonalising pseudopotentials (OPP) for removing Pauli forbidden states in the 3-body system within variational approach on Gaussian basis (V. I. Kukulin and V. N. Pomerantsev, Ann. Phys. (N.Y.) **111**, 330 (1978))

$$\tilde{H} = H_0 + \tilde{V}(r_{12}) + \tilde{V}(r_{23}) + \tilde{V}(r_{31})$$

- Hamiltonian.

$$\tilde{V}(r_{ij}) = V(r_{ij}) + \sum_f \lambda_f \hat{\Gamma}_{ij}^{(f)}.$$

- pseudopotentials.

$$\Psi_s^{JM} = \sum_{\gamma j} c_j^{(\lambda,l)} \varphi_{\gamma j}^s,$$

- expansion on symmetrized Gaussian basis.

$$\varphi_{\gamma j}^s = \varphi_{\gamma j}(1;2,3) + \varphi_{\gamma j}(2;3,1) + \varphi_{\gamma j}(3;1,2),$$

$$\varphi_{\gamma j}(k;l,m) = N_j^{(\lambda l)} x_k^\lambda y_k^l \exp\left(-\alpha_{\lambda j} x_k^2 - \beta_{lj} y_k^2\right) \times \mathcal{F}_{\lambda l}^{JM}(\mathbf{x}_k, \mathbf{y}_k).$$

$$\mathbf{x}_k = \frac{\sqrt{\mu}}{\hbar}\left(\mathbf{r}_l - \mathbf{r}_m\right) \equiv \tau^{-1}\mathbf{r}_{l,m}$$

$$\mathbf{y}_k = \frac{2\sqrt{\mu}}{\sqrt{3}\hbar}\left(\frac{\mathbf{r}_l + \mathbf{r}_m}{2} - \mathbf{r}_k\right) \equiv \tau_1^{-1}\boldsymbol{\rho}_k$$

- Jacobi coordinates

$$\alpha_{\lambda j} = \alpha_0 \tan\left(\frac{2j-1}{2N_\lambda}\frac{\pi}{2}\right), \quad j = 1, 2, \ldots N_\lambda$$

$$\beta_{lj} = \beta_0 \tan\left(\frac{2j-1}{2N_l}\frac{\pi}{2}\right), \quad j = 1, 2, \ldots N_l,$$

- Chebishev grid

$$\mathcal{F}_{\lambda l}^{JM}\left(\widehat{\mathbf{x}_k}, \widehat{\mathbf{y}_k}\right) = \left\{Y_\lambda\left(\widehat{\mathbf{x}_k}\right) \otimes Y_l\left(\widehat{\mathbf{y}_k}\right)\right\}_{JM} \phi(1)\,\phi(2)\,\phi(3)$$

- angular part of the basis w.f.

$$\hat{H} = -\frac{\partial^2}{\partial \mathbf{x}_k^2} - \frac{\partial^2}{\partial \mathbf{y}_k^2}$$

- kinetic energy operator

Projector on the fixed f -wave in the i- two-body system:

$$\hat{\Gamma}_i^{(f)} = \frac{1}{2f+1} \times \sum_{m_f} \left|\varphi_{fm_f}(\mathbf{x}_i)\right\rangle \left\langle \varphi_{fm_f}\left(\mathbf{x}'_i\right)\right| \delta\left(\mathbf{y}_i - \mathbf{y}'_i\right)$$

Complete 3-body projector, proven: kernel (Γ)=kernel (P)

$$\hat{P}_i = \sum_f \hat{\Gamma}_i^{(f)},$$

$$\hat{P} = \sum_{i=1}^{3} \hat{P}_i,$$

$$\hat{\Gamma} = \sum_{i=1}^{3} \hat{P}_i - \sum_{i \neq j=1}^{3} \hat{P}_i \hat{P}_j + \sum_{i \neq j \neq k=1}^{3} \hat{P}_i \hat{P}_j \hat{P}_k - \ldots .$$

In the OPP method with λ⟶+∞ the Pauli forbidden states should be removed from the solution of the 3-body Schroedinger equation.

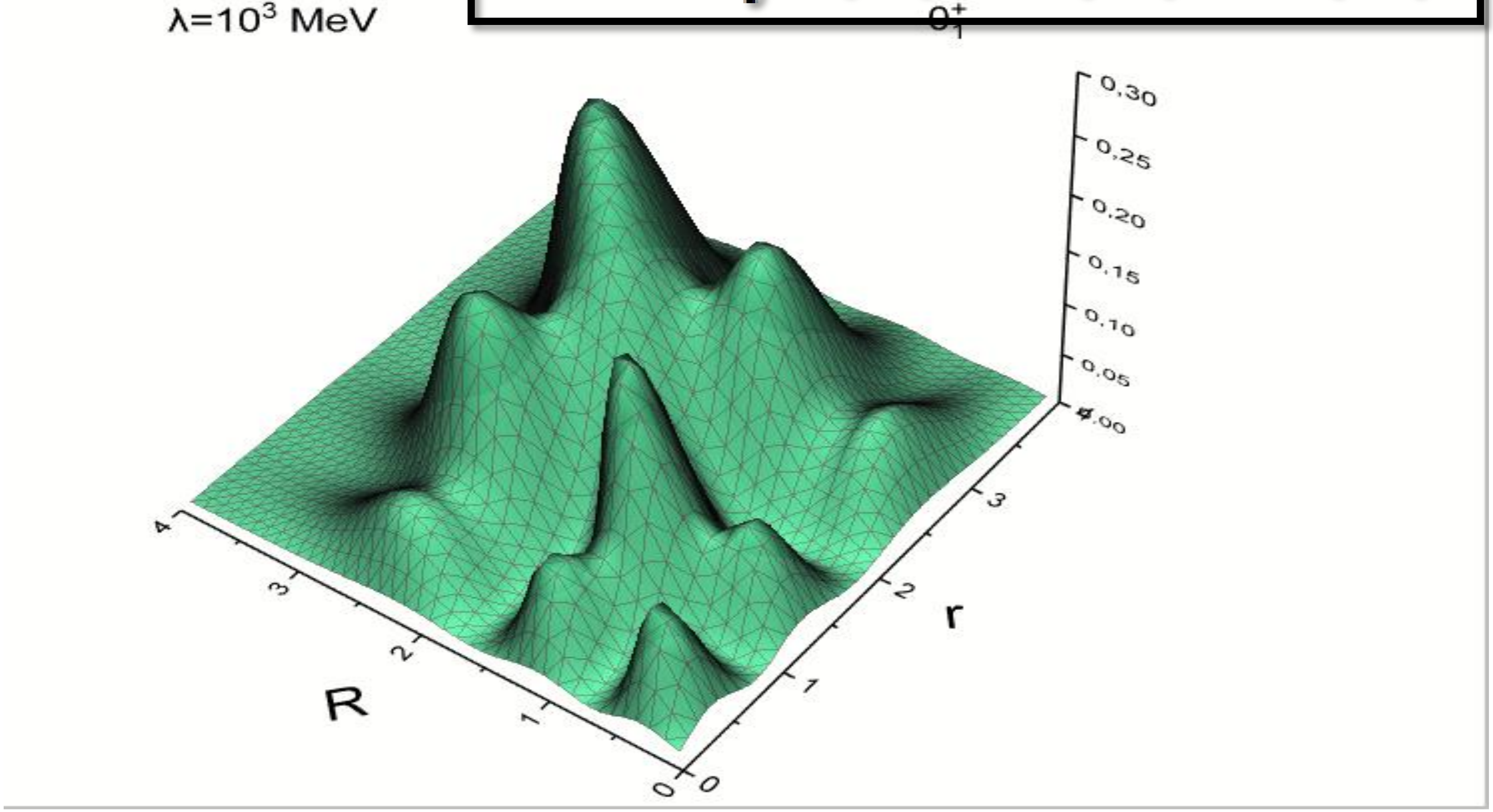
The energy spectrum of the 12*C* nucleus with (*Jπ; T*) = (0+; 0) in MeV at several values of the projecting constant

λ (MeV)	10	10^2	10^3	10^4	10^5	10^6	10^7	10^8
E_1	-210.69	-44.207	-20.15	-16.106	-.830	-0.435	-0.307	-0.283
E_2	-150.32	-17.585	-0.531	-0.422	1.353	1.407	1.513	1.551
E_3	-109.32	-15.310	+1.334	1.353	+3.019	3.316	4.038	4.055
$< P >$	29.95	28.92	1.130	3.777	0.721	8.76E-2	2.3E-2	4.7E-3

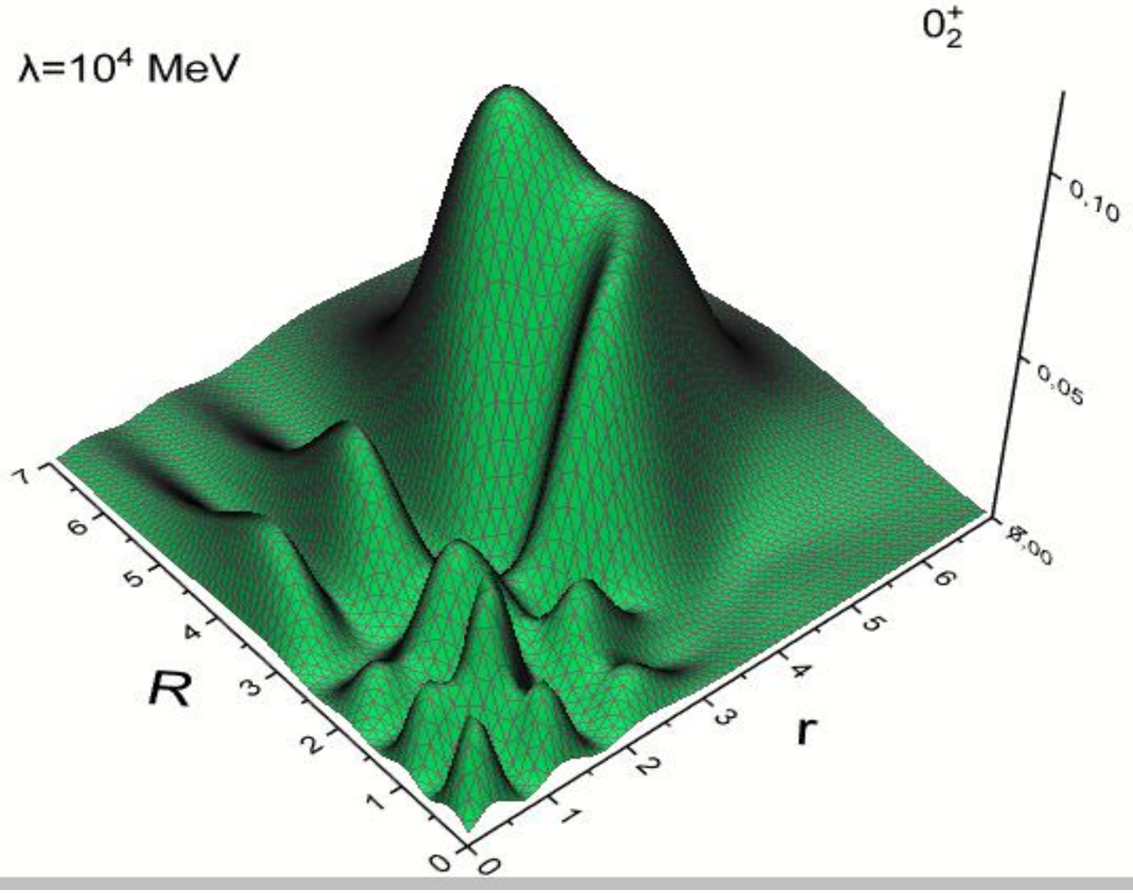
E. M. Tursunov, J. Phys. G: Nucl. Part. Phys. **27**, 1381 (2001).

Probability density function of matter for the ^{12}C(0+) g.s.

$$P(x,y)=\int d\hat{x}_1 d\hat{y}_1 \Psi^*(\vec{x}_1,\vec{y}_1)\Psi(\vec{x}_1,\vec{y}_1)$$



Probability density function of matter for the ^{12}C(0_2^+) (Hoyle) state



Contrbutions of 3-body partial waves to the energy of the ground and Hoyle states of the ^{12}C nucleus

λ	l	$N_\alpha N_\beta$	Contribution to g.s. (%)
0	0	14*12	28.777
2	2	14*12	35.722
4	4	16*14	34.885
6	6	16*14	0.606
8	8	16*14	0.010

$^{12}C\ (0_1^+)$ g.s.

λ	l	$N_\alpha N_\beta$	Contribution (%)
0	0	14*12	55.012
2	2	14*12	24.120
4	4	16*14	17.008
6	6	16*14	3.418
8	8	16*14	0.442

$^{12}C(0_2^+)$ Hoyle s.

The energy spectrum of the 12*C* nucleus with (*Jπ; T*) = (2+; 0) in MeV at several values of the projecting constant

λ (MeV)	10^3	10^4	10^5	10^6	10^7	10^8
E_1	-17.361	-15.649	-8.243	1.042	1.086	1.162
E_2	1.030	1.032	1.034	1.475	2.524	2.643
$< P >$	1.062	1.127	5.974	1.2E-2	3.1E-2	2.7E-2

Lowest 0^+ (g.s.) and 2_1^+ states go to the continuum when $\lambda \to +\infty$.

E. M. Tursunov, J. Phys. G: Nucl. Part. Phys. **27**, 1381 (2001).

B) DIRECT orthogonalization method (H. Matsumura, M. Orabi, Y. Suzuki, and Y. Fujiwara, Nucl. Phys. A **776**, 1 (2006)).

$$\hat{\Gamma}_i^{(f)} = \frac{1}{2f+1} \times \sum_{m_f} \left|\varphi_{fm_f}(\mathbf{x}_i)\right\rangle \left\langle\varphi_{fm_f}(\mathbf{x}'_i)\right| \delta\left(\mathbf{y}_i - \mathbf{y}'_i\right)$$

$$\hat{P} = \sum_{i=1}^{3} \hat{P}_i,$$

$$\hat{P}_i = \sum_{f} \hat{\Gamma}_i^{(f)},$$

The direct orthogonalization method is based on the separation of the complete Hilbert functional space into two parts. The first subspace ***L_Q***, which we call **allowed subspace**, is defined by the ***kernel of the complete three-body projector***. The rest subspace ***L_P*** contains 3α states forbidden by the Pauli principle. After the separation of the complete Hilbert functional space of 3α states into the L_Q (allowed) and ***L_P* (forbidden)** subspaces, at next step we solve the three-body Schrodinger equation in ***L_Q*.**

The allowed subspace L_Q is defined by the eigen states of the operator P, corresponding to its zero eigen value: $\boldsymbol{P\Phi = 0}$**.**

1 step: **Separation of** L_Q **= kern(*P*).**
2-step**: Solution of the Schroedinger equation in** L_Q.

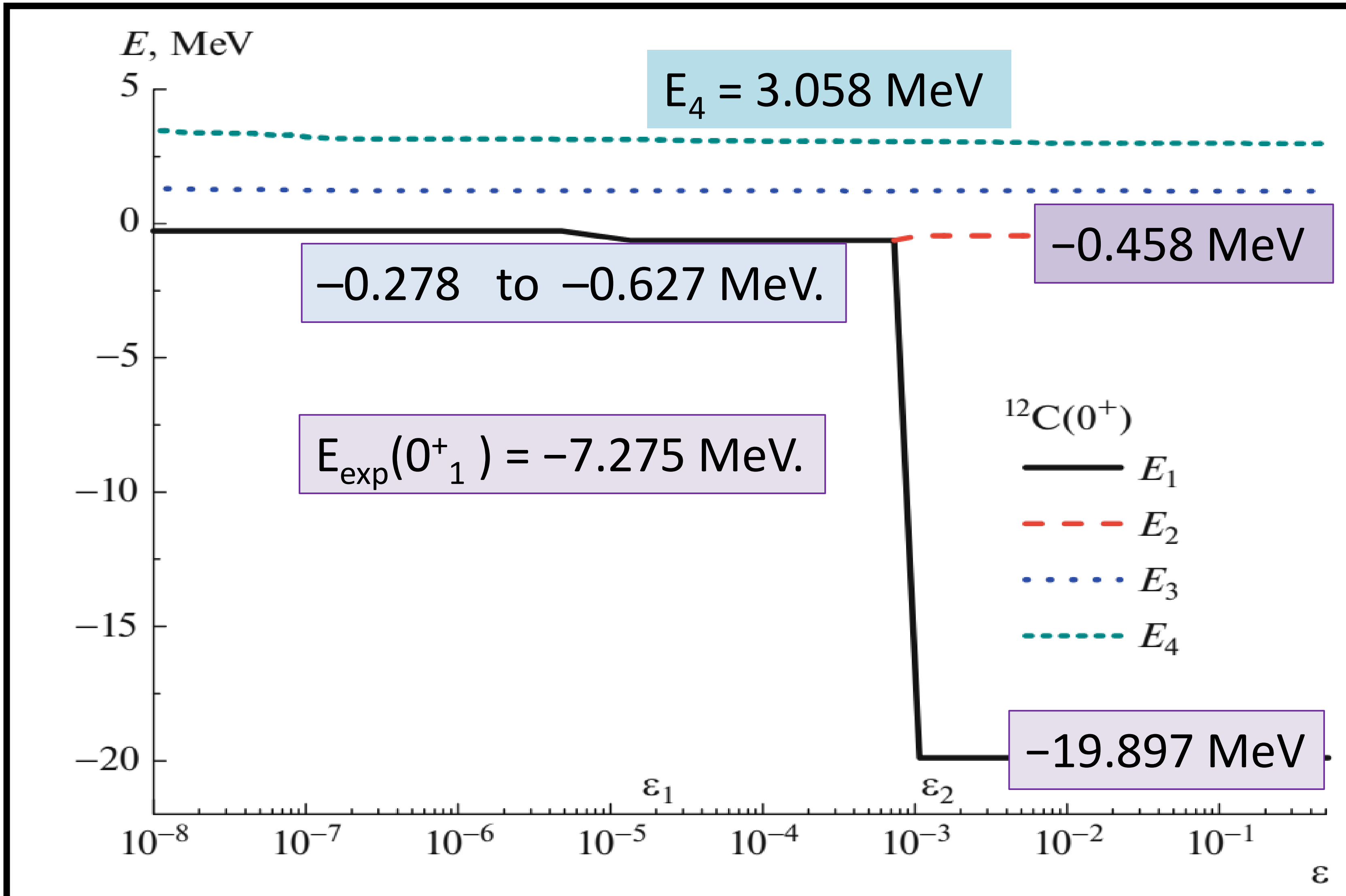
As in mentioned work, there are two eigen states of the operator ***P*** among other eigen states, which play a *decisive role* for the structure of the 12C(0+) lowest states. S***pecial (kritical) eigen states***

Φ_1: $\varepsilon_1 = 1.35333 \times 10^{-5}$ Φ_2: $\varepsilon_2 = 1.07152 \times 10^{-3}$

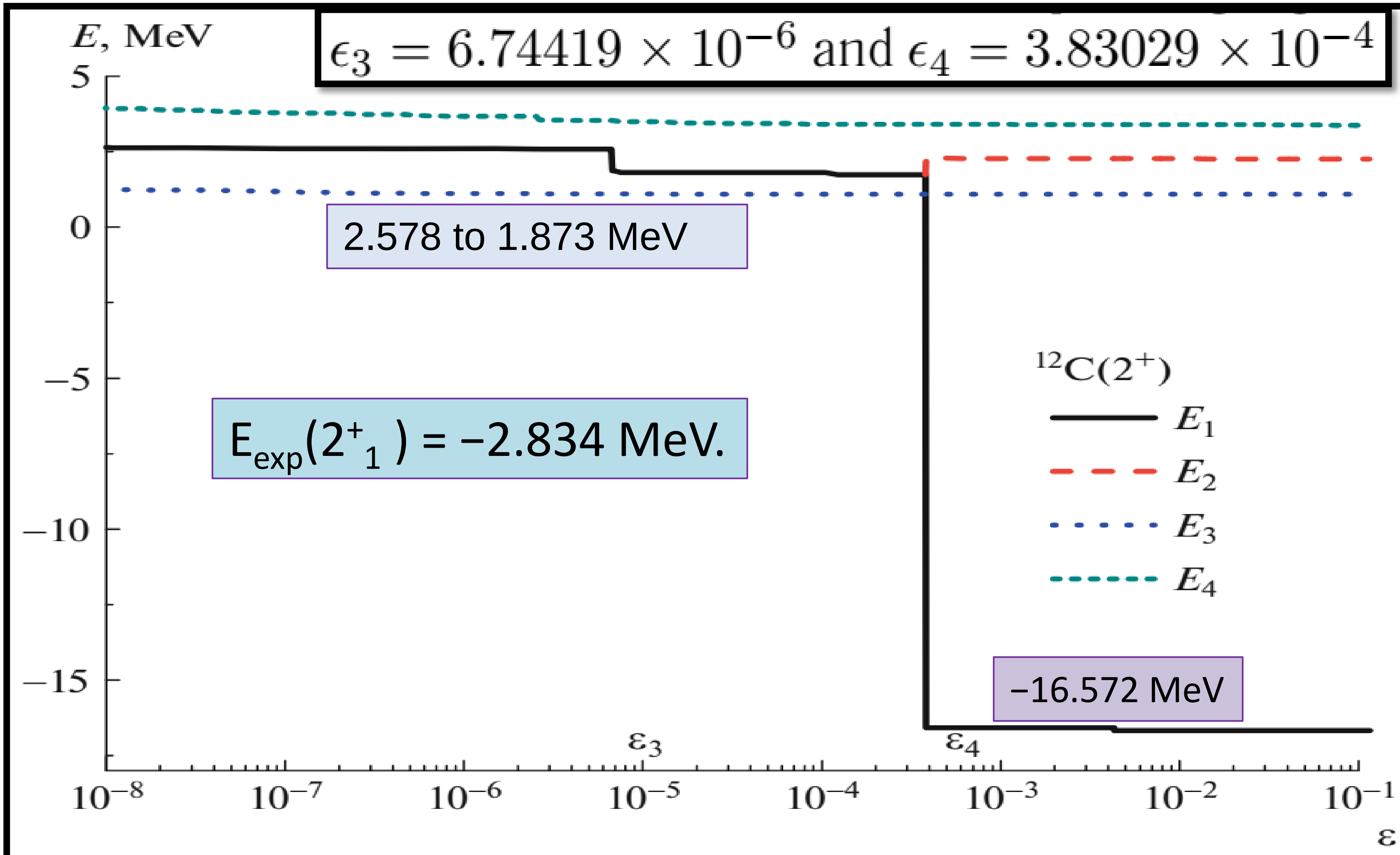
$$P\,\Phi_k = \varepsilon_k \Phi_k$$

In the paper of H. Matsumura et all they are called "almost forbidden states".

Energy spectrum of the lowest 12C(0+) states in dependence on the maximally allowed eigen value of the operator ***P***



Energy spectrum of the lowest 12C(2+) states in dependence on the maximally allowed eigen values of the operator ***P***



Possible first order quantum phase transition !?

From the weakly bound phase to a deep phase.
For the 0+ spectrum: *a critical eigen function (critical point)* and corresponding *critical eigen value of the threebody projector*, which is responsible for the quantum phase transition.
On the left-hand side of the critical point the lowest 0+ state mostly presents the Hoyle state, while *on the right-hand side* of the critical point the lowest state *becomes the ground state of the 12C nucleus in the deep phase*. An overlap of the critical eigen function of the three-body projector with the ground state is close to unity, while its overlap with the Hoyle state is almost zero. This means that the ***ground state of the 12C nucleus in the deep phase is created by the critical eigen function of the Pauli projector***.
A behavior of the 2+ levels is analogous.

Nuclear Binding Near a Quantum Phase Transition

PRL 117, 132501 (2016)

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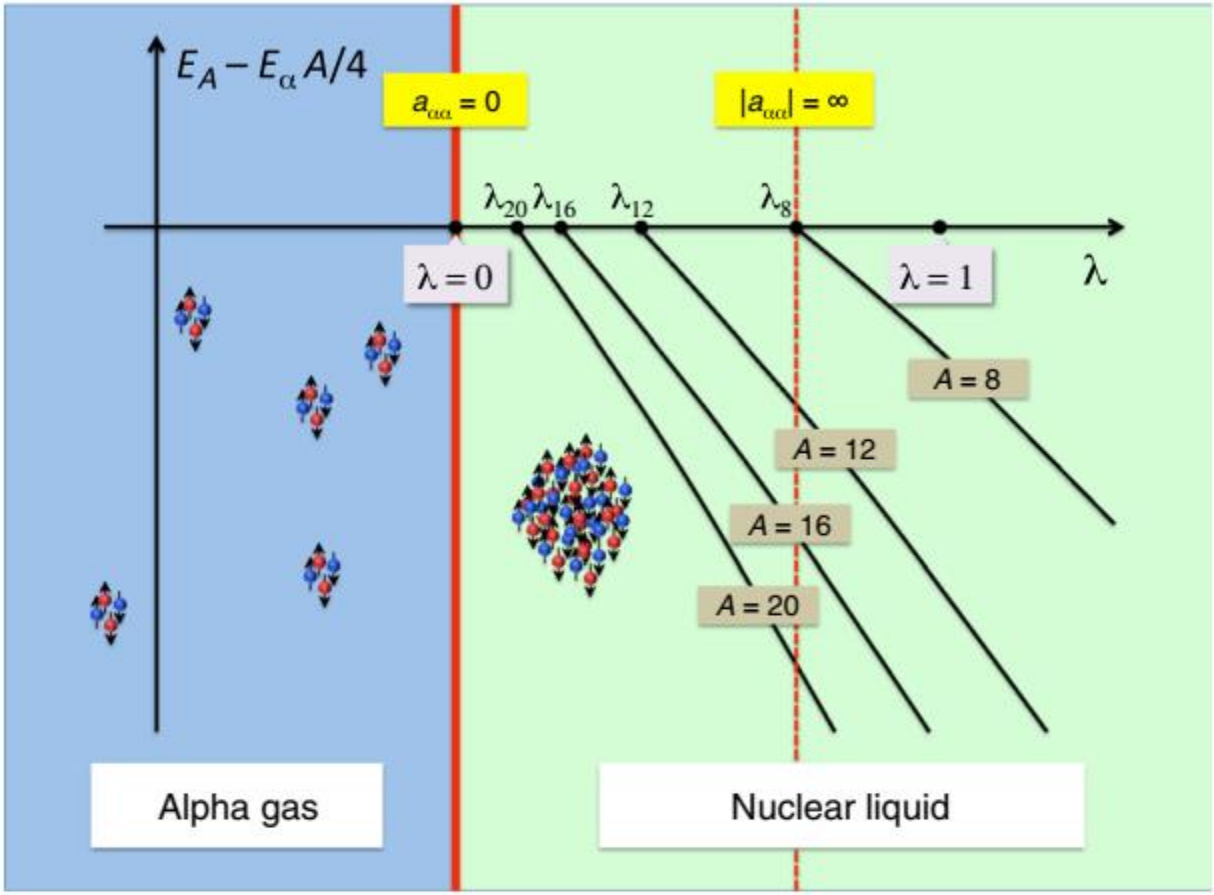

$$V_\lambda = (1-\lambda)V_A + \lambda V_B$$

λ - strength of the locality of the ChEFT NN-interaction pot-al.

While the properties of few nucleon systems vary only slightly with λ, the many-body ground state of V_λ undergoes a ***quantum phase transition from a Bose condensed gas to a nuclear liquid.***

Conclusions

- The energy spectrum of the ^{12}C nucleus has been analyzed within the 3α model. The Pauli forbidden states were treated by the OPP and exact orthogonalisation methods.
- An evidence of possible first order quantum phase transition has been examined. It was shown that there are effects of possible QPT in the lowest $^{12}C(0^+_1)$ and $^{12}C(2^+_1)$ states from the weakly bound *gas-like phase* to a *deep liquid-like* phase.
- There is a *critical eigen function and corresponding critical eigen value of the 3-body Pauli projector* which are responsible for the quantum phase transition.
- On the left-hand side of the critical point the lowest 0^+ state mostly presents the *Hoyle state,* while on the *right-hand side* of the critical point the lowest state becomes *the ground state of the ^{12}C nucleus in the deep phase.*
- *The same behavior was found in the 2^+ sector.*

THANKS !!!