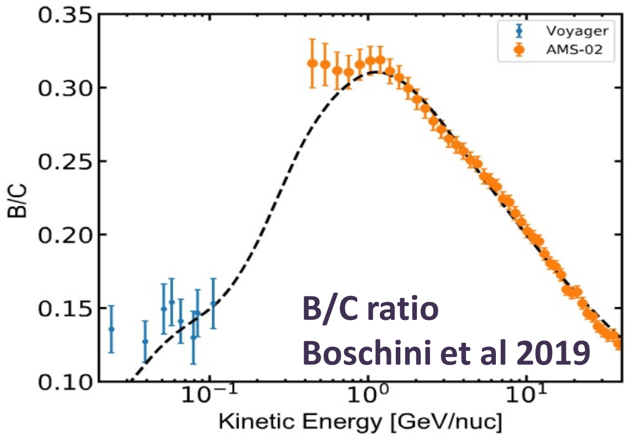
ПКЛ-1, ID-132

On the diffusion of cosmic rays with a back reaction on the cascade of magnetosonic waves in the interstellar medium

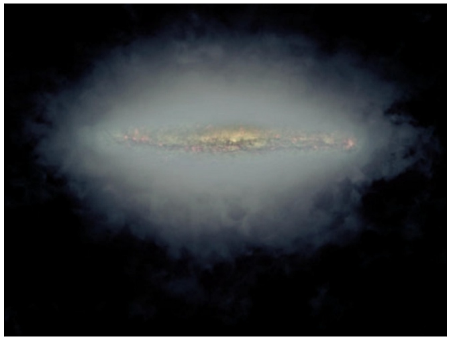
V. S. Ptuskin [1], V. N. Zirakashvili [1],
I. V. Moskalenko [2], E. S. Seo[3]

[1] IZMIRAN, Troitsk, Moscow
[2] Stanford University, Stanford, USA
[3] University of Maryland, College Park, USA

peak in secondary-to-primary ratios at ~ 1 GeV/nuc reflects some feature of cosmic ray propagation in the interstellar medium



B/C ratio
Boschini et al 2019



sec

prim

$$\frac{J_2}{T_e} \approx n v \sigma_{12} J_1$$

leakage time

CR transport equation:

Berezinskii et al 1990, Strong et al 2007

$$\frac{\partial\Psi}{\partial t} - \nabla(\mathbf{D}\nabla\Psi) + \nabla(\mathbf{u}\Psi) + \frac{\partial}{\partial p}\left(\left(\frac{dp}{dt}\right)_{\text{loss}}\Psi\right) - \frac{\nabla\mathbf{u}}{3}\frac{\partial}{\partial p}(pF)\Psi - \frac{\partial}{\partial p}\left(p^2 D_{pp}\frac{\partial}{\partial p}\frac{\Psi}{p^2}\right) + \frac{\Psi}{\tau_{\text{cat}}} = Q$$

diffusion

convection

energy loss, <0

adiabatic change of momentum

$$\dot{p} = -\frac{\nabla\mathbf{u}}{3}p$$

stochastic acceleration

catastrophic losses (fragmentation, decay ...)

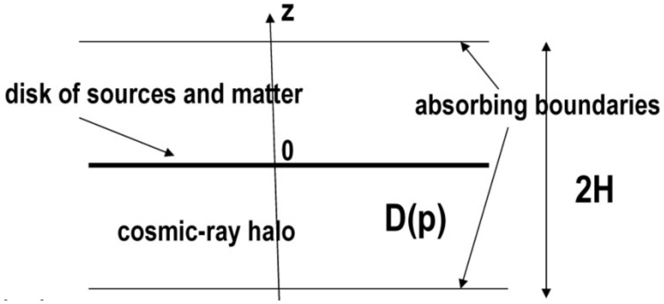
source term

number density $N_{cr} = \int dp\Psi$

intensity $J(E) = \Psi(p)/4\pi$

simple 1D model

$$Q \propto \delta(z),\ n \propto \delta(z),\ u = 0$$



equation for stable CR nuclei:

Ptuskin, Soutoul 1990, Seo, Ptuskin 1994, Jones et al 2001, …

$$\bigstar\quad \frac{\Psi}{X_e} + \frac{\sigma}{m_{ism}}\Psi + \ldots = \frac{q}{\mu v}, \qquad X_e = \frac{\mu v H}{2D}$$

- escape length

energy loss and reacceleration

surface gas density

Cosmic ray diffusion

wave-particle resonant interaction $r_g \sim 1/k$

Larmor radius

wave number

Jokipii 1966 …

diffusion coefficient $$D \approx \frac{v r_g}{3}\frac{B^2}{\delta B^2_{res}}, \quad \frac{\delta B^2}{4\pi} = \int dk W_k$$

diffusion on momentum $$D_{pp} \propto \frac{V_A^2}{D} p^2$$

energy density of MHD turbulence

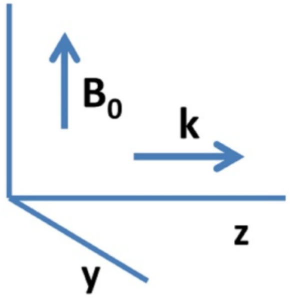
Alfven and magnetosonic waves

$$\mathbf{A}: \ \omega = k_z V_a$$

$$u_y, \delta B_y \neq 0,$$

$$\mathbf{MS}: \ \omega = k V_\pm$$

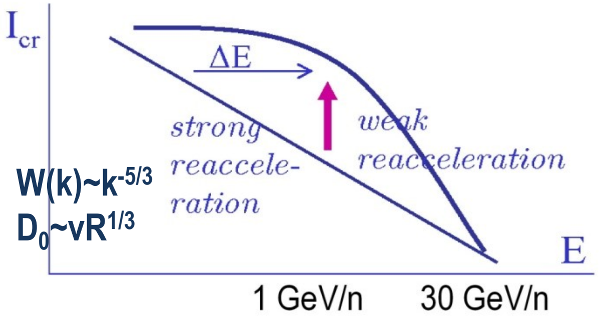
$$\delta\rho, u_x, u_y, \delta B_x \neq 0.$$



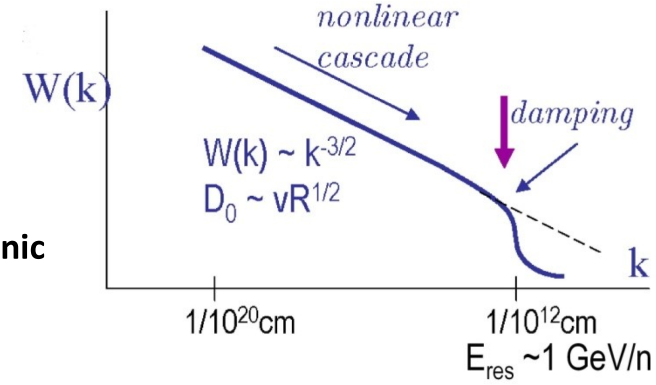
Possible nature of sec/prim peaks

Seo, Ptuskin 1994, Ptuskin et al 2006

Kolmogorov type nonlinear cascade of Alfven waves



Iroshnikov-Kraichnan Nonlinear cascade of magnetosonic waves



Problems:

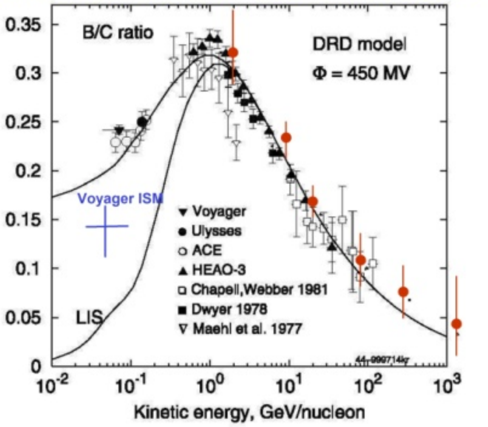
- wave distribution is strongly anisotropic, ($k_{\|} = k_{\perp}^{2/3} L^{-1/3}$ Goldreich, Sridhar 1995) and CR scattering is not efficient
- difficulty with interpretation of radioastronomical data Orlando 2018

- heavy damping in the interstellar medium but CR scattering can be provided in Galactic halo Yan, Lazarian 2002, Xu, Lazarian 2020
- scattering is too weak below 1 GV in our model Ptuskin et al 2006

Equation for waves.

Ptuskin et al 2006

diffusion (Kraichnan) + reac. + damping on CR



our equation (2006) with reference to Chandrasekhar 1948 and Heisenberg 1948

source at long wavelength

$$\boxed{\frac{d}{dk}\left[\frac{C_M}{\rho V_A}k^3W_k^2\right]} = -2\Gamma_{cr}W_k + S\delta(k-k_L)$$

damping on CR

$$= \frac{\pi e^2 V_A^2}{2kc^2}\int_{p_{res}(k)} dp\,p^{-1}\Psi(p)$$

new equation

$$\underbrace{\frac{d}{dk}\left[\frac{C_M^*}{\rho V_A}kW_k\int^k dk_1 k_1 W_{k_1}\right]} = -2\Gamma_{cr}W_k + S\delta(k-k_L) \quad ★★$$

Evaluation formula for nonlinear cascade of MS wave derived from the wave kinetic equations for plasmons with resonant three-wave interactions (Sagdeev, Zakharov (1970), Akhiezer et al (1974), Chashey, Shishov (1985), Zirakashvili (2000) , Chandran (2005)). We hold leading terms that describe efficient energy transfer to short waves.

Analytical solution of Eqs. (★), (★★) without CR ionization energy losses gives

$$D = D_0 \cdot \frac{(kk_L)^{-1/2}\int^k dk_1\left(1+\frac{\alpha_{k1}}{2}\right)}{\exp\left[\frac{1}{2}\int_{k_L}^{k}\frac{dk_2}{\int^{k_2} dk_1\left(1+\frac{\alpha_{k1}}{2}\right)}\right]}, \quad \text{where } \alpha_{\mathrm{k}} = \frac{3\pi e^2 V_A H q_0(p)}{2C_M^* c^2 \mathrm{v}k}, \quad k = \frac{1}{r_g}.$$

resonance

diffusion coefficient without effect of damping

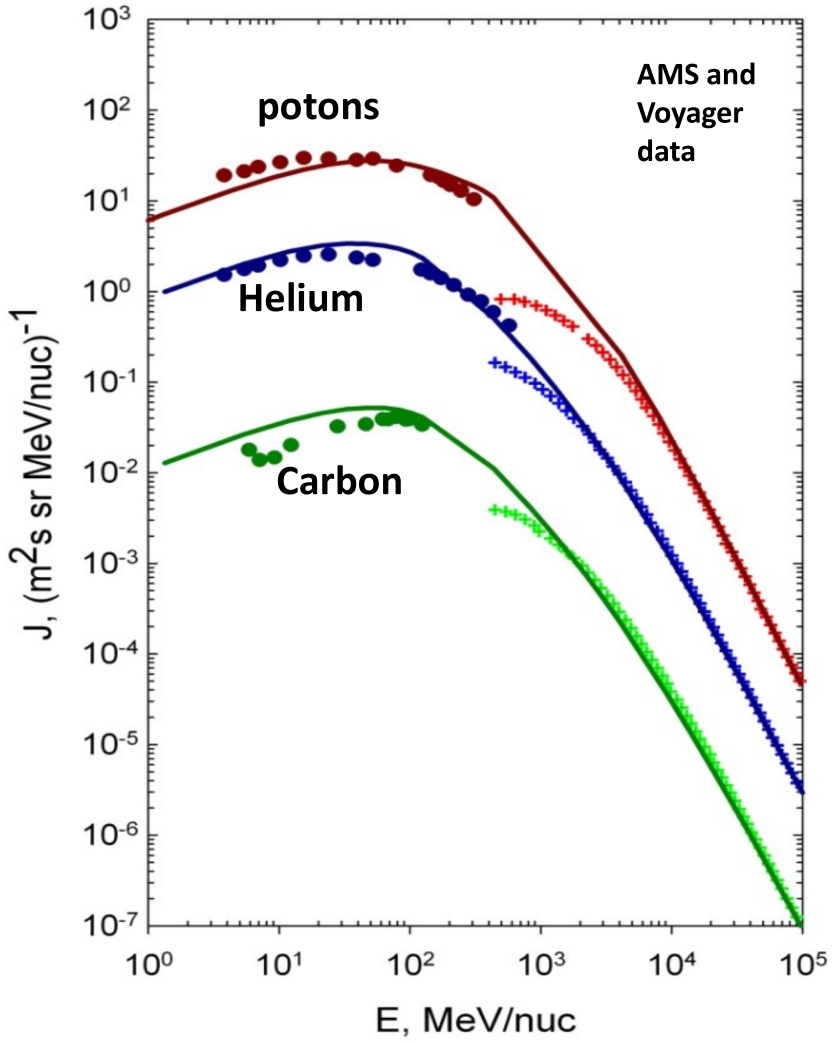
$D_0 \sim vr_g^{1/2}$

parameter $\sim \dfrac{\mathbf{eD(p)N_{cr}(>p)}}{\mathbf{C_M^*\sqrt{\rho cv}}}$

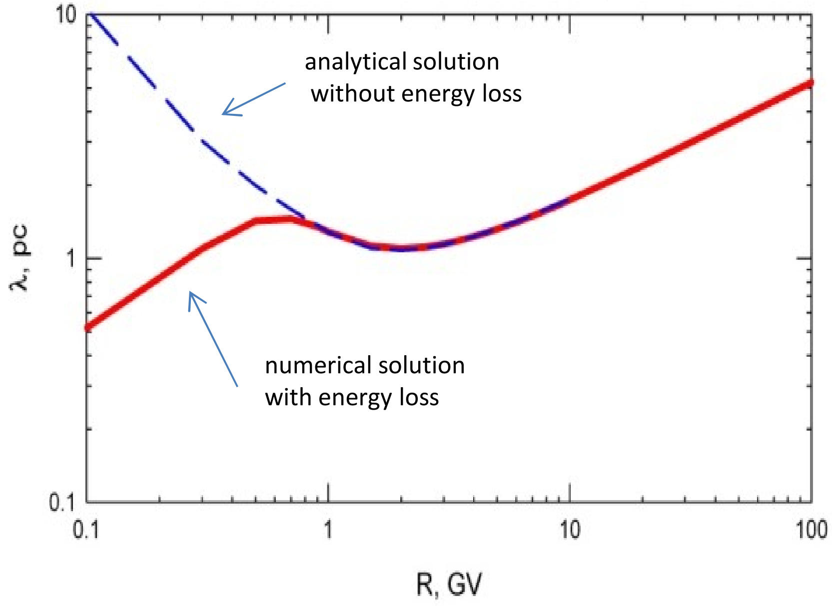
determines spectrum distortion

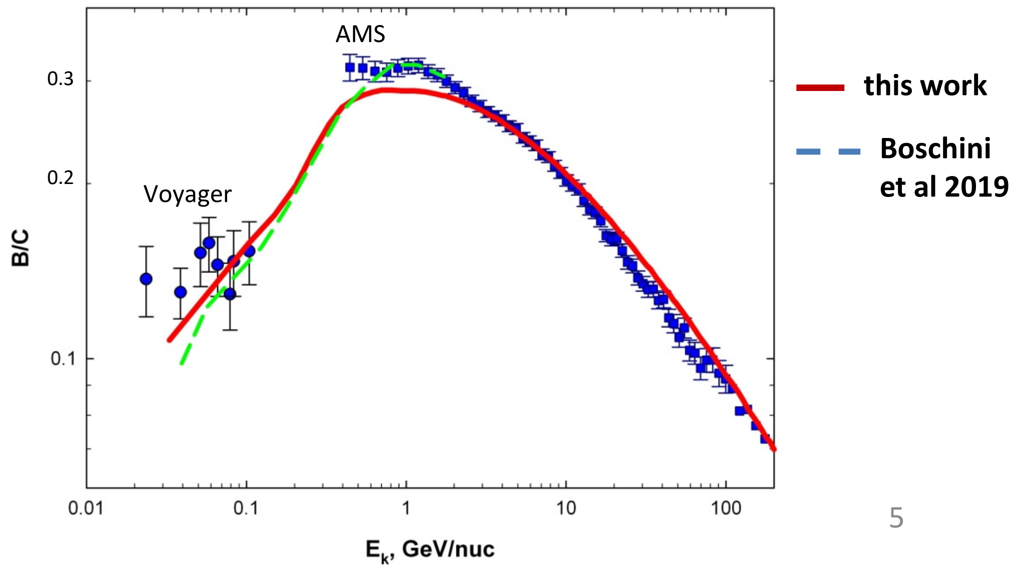
Results of calculations

Accepted values:
- CR source : q_p~$R^{-1.4}$ at < 1GV, $R^{-2.4}$ at >1GV, $q_{nuc}=q_p\ R^{0.15}$
- halo parameters: H=4 kpc, n= 6 10^{-3} cm^{-3} , B= 5 10^{-6} G
- surface gas density of galactic disk 2.4 mg/cm^2



diffusion mean free path $\lambda = \frac{3}{\mathrm{v}} \cdot D$





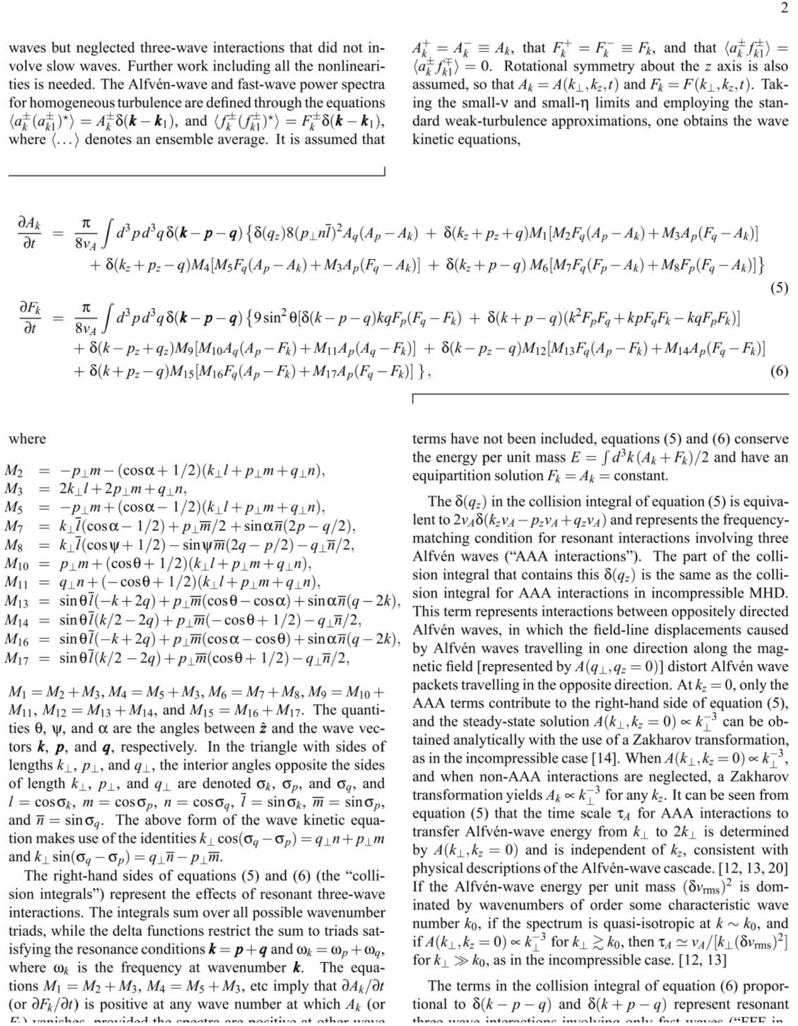
Conclusion

Damping of interstellar turbulence on cosmic rays remains a possible explanation for the observed energy dependence of cosmic-ray diffusion in the Galaxy below several GeV/n. New estimative equation for magnetosonic turbulence allows extending the original propagation model with damping to MeV energies. The cosmic ray transport equation is solved simultaneously with this equation. Calculations demonstrate reasonable agreement of the theory with observations of primary and secondary nuclei in the energy range 1 MeV/n to 100 GeV/n although some model improvement is required.

Appendix

theory of MHD turbulence

B. Chandran 2005 Phys. Rev. L. 95, 5004



waves but neglected three-wave interactions that did not involve slow waves. Further work including all the nonlinearities is needed. The Alfvén-wave and fast-wave power spectra for homogeneous turbulence are defined through the equations $\langle a_k^\pm (a_{k1}^\pm)^\star \rangle = A_k^\pm \delta(\boldsymbol{k}-\boldsymbol{k}_1)$, and $\langle f_k^\pm (f_{k1}^\pm)^\star \rangle = F_k^\pm \delta(\boldsymbol{k}-\boldsymbol{k}_1)$, where $\langle \ldots \rangle$ denotes an ensemble average. It is assumed that $A_k^+ = A_k^- \equiv A_k$, that $F_k^+ = F_k^- \equiv F_k$, and that $\langle a_k^\pm f_{k1}^\pm \rangle = \langle a_k^\pm f_{k1}^\mp \rangle = 0$. Rotational symmetry about the z axis is also assumed, so that $A_k = A(k_\perp, k_z, t)$ and $F_k = F(k_\perp, k_z, t)$. Taking the small-ν and small-η limits and employing the standard weak-turbulence approximations, one obtains the wave kinetic equations,

$$\begin{aligned}\frac{\partial A_k}{\partial t} &= \frac{\pi}{8v_A}\int d^3p\,d^3q\,\delta(\boldsymbol{k}-\boldsymbol{p}-\boldsymbol{q})\,\{\delta(q_z)8(p_\perp n\bar{l})^2 A_q(A_p-A_k) + \delta(k_z+p_z+q)M_1[M_2F_q(A_p-A_k)+M_3A_p(F_q-A_k)]\\ &\quad + \delta(k_z+p_z-q)M_4[M_5F_q(A_p-A_k)+M_3A_p(F_q-A_k)] + \delta(k_z+p-q)\,M_6[M_7F_q(F_p-A_k)+M_8F_p(F_q-A_k)]\}\end{aligned} \qquad (5)$$

$$\begin{aligned}\frac{\partial F_k}{\partial t} &= \frac{\pi}{8v_A}\int d^3p\,d^3q\,\delta(\boldsymbol{k}-\boldsymbol{p}-\boldsymbol{q})\,\{9\sin^2\theta[\delta(k-p-q)kqF_p(F_q-F_k) + \delta(k+p-q)(k^2F_pF_q+kpF_qF_k-kqF_pF_k)]\\ &\quad + \delta(k-p_z+q_z)M_9[M_{10}A_q(A_p-F_k)+M_{11}A_p(A_q-F_k)] + \delta(k-p_z-q)M_{12}[M_{13}F_q(A_p-F_k)+M_{14}A_p(F_q-F_k)]\\ &\quad + \delta(k+p_z-q)M_{15}[M_{16}F_q(A_p-F_k)+M_{17}A_p(F_q-F_k)]\,\},\end{aligned} \qquad (6)$$

where

$$\begin{aligned}M_2 &= -p_\perp m-(\cos\alpha+1/2)(k_\perp l+p_\perp m+q_\perp n),\\ M_3 &= 2k_\perp l+2p_\perp m+q_\perp n,\\ M_5 &= -p_\perp m+(\cos\alpha-1/2)(k_\perp l+p_\perp m+q_\perp n),\\ M_7 &= k_\perp\bar{l}(\cos\alpha-1/2)+p_\perp\overline{m}/2+\sin\alpha\,\overline{n}(2p-q/2),\\ M_8 &= k_\perp\bar{l}(\cos\psi+1/2)-\sin\psi\,\overline{m}(2q-p/2)-q_\perp\overline{n}/2,\\ M_{10} &= p_\perp m+(\cos\theta+1/2)(k_\perp l+p_\perp m+q_\perp n),\\ M_{11} &= q_\perp n+(-\cos\theta+1/2)(k_\perp l+p_\perp m+q_\perp n),\\ M_{13} &= \sin\theta\,\bar{l}(-k+2q)+p_\perp\overline{m}(\cos\theta-\cos\alpha)+\sin\alpha\,\overline{n}(q-2k),\\ M_{14} &= \sin\theta\,\bar{l}(k/2-2q)+p_\perp\overline{m}(-\cos\theta+1/2)-q_\perp\overline{n}/2,\\ M_{16} &= \sin\theta\,\bar{l}(-k+2q)+p_\perp\overline{m}(\cos\alpha-\cos\theta)+\sin\alpha\,\overline{n}(q-2k),\\ M_{17} &= \sin\theta\,\bar{l}(k/2-2q)+p_\perp\overline{m}(\cos\theta+1/2)-q_\perp\overline{n}/2,\end{aligned}$$

$M_1 = M_2+M_3$, $M_4 = M_5+M_3$, $M_6 = M_7+M_8$, $M_9 = M_{10}+M_{11}$, $M_{12} = M_{13}+M_{14}$, and $M_{15} = M_{16}+M_{17}$. The quantities θ, ψ, and α are the angles between $\hat{\boldsymbol{z}}$ and the wave vectors $\boldsymbol{k}$, $\boldsymbol{p}$, and $\boldsymbol{q}$, respectively. In the triangle with sides of lengths $k_\perp$, $p_\perp$, and $q_\perp$, the interior angles opposite the sides of length $k_\perp$, $p_\perp$, and $q_\perp$ are denoted σ_k, σ_p, and σ_q, and $l = \cos\sigma_k$, $m = \cos\sigma_p$, $n = \cos\sigma_q$, $\bar{l} = \sin\sigma_k$, $\overline{m} = \sin\sigma_p$, and $\overline{n} = \sin\sigma_q$. The above form of the wave kinetic equation makes use of the identities $k_\perp\cos(\sigma_q-\sigma_p) = q_\perp n+p_\perp m$ and $k_\perp\sin(\sigma_q-\sigma_p) = q_\perp\overline{n}-p_\perp\overline{m}$.

The right-hand sides of equations (5) and (6) (the "collision integrals") represent the effects of resonant three-wave interactions. The integrals sum over all possible wavenumber triads, while the delta functions restrict the sum to triads satisfying the resonance conditions $\boldsymbol{k} = \boldsymbol{p}+\boldsymbol{q}$ and $\omega_k = \omega_p+\omega_q$, where ω_k is the frequency at wavenumber $\boldsymbol{k}$. The equations $M_1 = M_2+M_3$, $M_4 = M_5+M_3$, etc imply that $\partial A_k/\partial t$ (or $\partial F_k/\partial t$) is positive at any wave number at which A_k (or F_k) vanishes, provided the spectra are positive at other wave

terms have not been included, equations (5) and (6) conserve the energy per unit mass $E = \int d^3k\,(A_k+F_k)/2$ and have an equipartition solution $F_k = A_k =$ constant.

The $\delta(q_z)$ in the collision integral of equation (5) is equivalent to $2v_A\delta(k_zv_A-p_zv_A+q_zv_A)$ and represents the frequency-matching condition for resonant interactions involving three Alfvén waves ("AAA interactions"). The part of the collision integral that contains this $\delta(q_z)$ is the same as the collision integral for AAA interactions in incompressible MHD. This term represents interactions between oppositely directed Alfvén waves, in which the field-line displacements caused by Alfvén waves travelling in one direction along the magnetic field [represented by $A(q_\perp, q_z = 0)$] distort Alfvén wave packets travelling in the opposite direction. At $k_z = 0$, only the AAA terms contribute to the right-hand side of equation (5), and the steady-state solution $A(k_\perp, k_z = 0) \propto k_\perp^{-3}$ can be obtained analytically with the use of a Zakharov transformation, as in the incompressible case [14]. When $A(k_\perp, k_z = 0) \propto k_\perp^{-3}$, and when non-AAA interactions are neglected, a Zakharov transformation yields $A_k \propto k_\perp^{-3}$ for any k_z. It can be seen from equation (5) that the time scale τ_A for AAA interactions to transfer Alfvén-wave energy from $k_\perp$ to $2k_\perp$ is determined by $A(k_\perp, k_z = 0)$ and is independent of k_z, consistent with physical descriptions of the Alfvén-wave cascade. [12, 13, 20] If the Alfvén-wave energy per unit mass $(\delta v_{\rm rms})^2$ is dominated by wavenumbers of order some characteristic wave number k_0, if the spectrum is quasi-isotropic at $k \sim k_0$, and if $A(k_\perp, k_z = 0) \propto k_\perp^{-3}$ for $k_\perp \gtrsim k_0$, then $\tau_A \simeq v_A/[k_\perp(\delta v_{\rm rms})^2]$ for $k_\perp \gg k_0$, as in the incompressible case. [12, 13]

The terms in the collision integral of equation (6) proportional to $\delta(k-p-q)$ and $\delta(k+p-q)$ represent resonant

empirical modeling of CR propagation (GALPROP code)

Inference of the Local Interstellar Spectra of Cosmic Ray Nuclei Z < 28 with the GALPROP–HELMOD Framework

M. J. BOSCHINI,1, 2 S. DELLA TORRE,1 M. GERVASI,1, 3 D. GRANDI,1, 3 G. J´OHANNESSON,4, 5 G. LA VACCA,1, 3 N. MASI,6, 7 I. V. MOSKALENKO,8, 9 S. PENSOTTI,1, 3 T. A. PORTER,8, 9 L. QUADRANI,6, 7 P. G. RANCOITA,1 D. ROZZA,1, 3 AND M. TACCONI1, 3

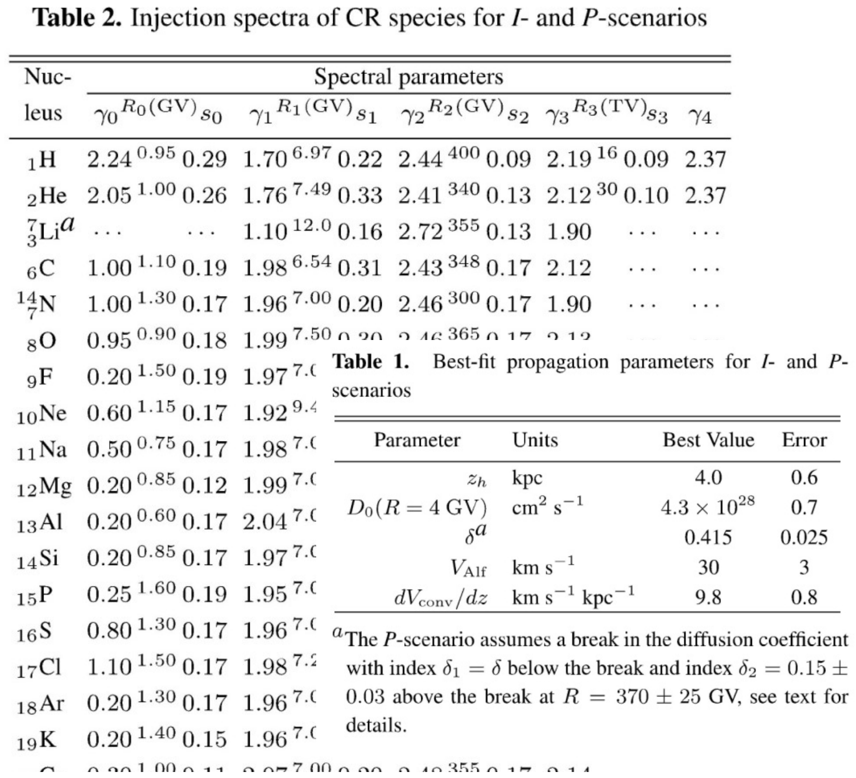


Table 2. Injection spectra of CR species for *I*- and *P*-scenarios

Nucleus	Spectral parameters												
	γ_0	R_0(GV)	s_0	γ_1	R_1(GV)	s_1	γ_2	R_2(GV)	s_2	γ_3	R_3(TV)	s_3	γ_4
$_1$H	2.24	0.95	0.29	1.70	6.97	0.22	2.44	400	0.09	2.19	16	0.09	2.37
$_2$He	2.05	1.00	0.26	1.76	7.49	0.33	2.41	340	0.13	2.12	30	0.10	2.37
$^7_3$Li^a	…		…	1.10	12.0	0.16	2.72	355	0.13	1.90		…	…
$_6$C	1.00	1.10	0.19	1.98	6.54	0.31	2.43	348	0.17	2.12		…	…
$^{14}_7$N	1.00	1.30	0.17	1.96	7.00	0.20	2.46	300	0.17	1.90		…	…
$_8$O	0.95	0.90	0.18	1.99	7.50	[illegible]	[illegible]	365	[illegible]	[illegible]			
$_9$F	0.20	1.50	0.19	1.97	[illegible]								
$_{10}$Ne	0.60	1.15	0.17	1.92	[illegible]								
$_{11}$Na	0.50	0.75	0.17	1.98	[illegible]								
$_{12}$Mg	0.20	0.85	0.12	1.99	[illegible]								
$_{13}$Al	0.20	0.60	0.17	2.04	[illegible]								
$_{14}$Si	0.20	0.85	0.17	1.97	[illegible]								
$_{15}$P	0.25	1.60	0.19	1.95	[illegible]								
$_{16}$S	0.80	1.30	0.17	1.96	[illegible]								
$_{17}$Cl	1.10	1.50	0.17	1.98	[illegible]								
$_{18}$Ar	0.20	1.30	0.17	1.96	[illegible]								
$_{19}$K	0.20	1.40	0.15	1.96	[illegible]								

Table 1. Best-fit propagation parameters for *I*- and *P*-scenarios

Parameter	Units	Best Value	Error
z_h	kpc	4.0	0.6
$D_0(R = 4\,\mathrm{GV})$	cm^2 s^{-1}	4.3×10^{28}	0.7
δ^a		0.415	0.025
$V_{\rm Alf}$	km s^{-1}	30	3
$dV_{\rm conv}/dz$	km s^{-1} kpc^{-1}	9.8	0.8

[a] The *P*-scenario assumes a break in the diffusion coefficient with index $\delta_1 = \delta$ below the break and index $\delta_2 = 0.15 \pm 0.03$ above the break at $R = 370 \pm 25$ GV, see text for details.